GEOMETRIZED VACUUM PHYSICS. PART V: STABLE VACUUM FORMATIONS

FÍSICA DEL VACÍO GEOMETRIZADO. PARTE V: FORMACIONES DE VACÍO ESTABLES

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ABSTRACT

This article is the fifth part of a scientific project under the general title "Geometrized vacuum physics based on the Algebra of Signatures". In this article, Einstein's vacuum equations are used as conservation laws, and their solutions as metric-dynamic models of stable vacuum formations. Sets of metrics-solutions of vacuum equations are considered, and methods of extracting information from these metrics based on Algebra of Signature are proposed. For convenience of perception of intra-vacuum processes, a change in the interpretation of the zero components of the metric tensor was used. Instead of curved space-time continua, "colored" elastoplastic continuous pseudo-mediums are introduced into consideration. In this case, the zero components of the metric tensor determine not the change in the rate of flow of local time, but the speed of flow of intra-vacuum current in the local region of the elastoplastic pseudo-medium. At the end of the article, an extended (third) Einstein vacuum equation is proposed, which allows us to consider metric-dynamic models of a variety of stable corpuscular vacuum formations. Alsigna's infinitely deepening intertwined fabric of space-time continuum, taking into account all 16 signatures (i.e. 16 types of topologies), is in many ways similar to the spin network of loop quantum gravity and to 6-dimensional Calabi-Yau manifolds. In this sense, the Algebra of Signatures can serve as a link that unites different directions in the development of quantum gravity.

RESUMEN

Este artículo es la quinta parte de un proyecto científico bajo el título general "*Física del vacío geometrizada basada en el Álgebra de Signatures*". En este artículo, las ecuaciones de vacío de Einstein se utilizan como leyes de conservación y sus soluciones como modelos métrico-dinámicos de formaciones de vacío estables. Se consideran conjuntos de soluciones métricas de ecuaciones de vacío y se proponen métodos para extraer información de estas métricas basados en el álgebra de firma. Para facilitar la percepción de los procesos intra-vacío, se utilizó un cambio en la interpretación de los componentes cero del tensor métrico. En lugar de continuos espacio-temporales curvos, se introducen en consideración pseudomedios continuos elastoplásticos "coloreados". En este caso, los componentes cero del tensor métrico determinan no el cambio en la velocidad del flujo del tiempo local, sino la velocidad del flujo de la corriente intra-vacío en la región local del pseudomedio elastoplástico. Al final del artículo, se propone una (tercera) ecuación de vacío de Einstein ampliada, que nos permite considerar modelos métrico-dinámicos de una variedad de formaciones de vacío corpusculares estables. El tejido entrelazado infinitamente cada vez más profundo de Alsigna del continuo espacio-tiempo, teniendo en cuenta las 16 firmas (es decir, 16 tipos de topologías), es en muchos aspectos similar a la red de espín de la gravedad cuántica de bucles y a las variedades de Calabi-Yau de 6 dimensiones. En este sentido, el Álgebra de Signatures puede servir como vínculo que une diferentes direcciones en el desarrollo de la gravedad cuántica.

Keywords: vacuum, vacuum equation, signature, algebra of signatures
Palabras clave: vacío, ecuación de vacío, firma, álgebra de signatures

BACKGROUND AND INTRODUCTION

"The best things in the world are not things"
Paraphrasing Art Buchwald

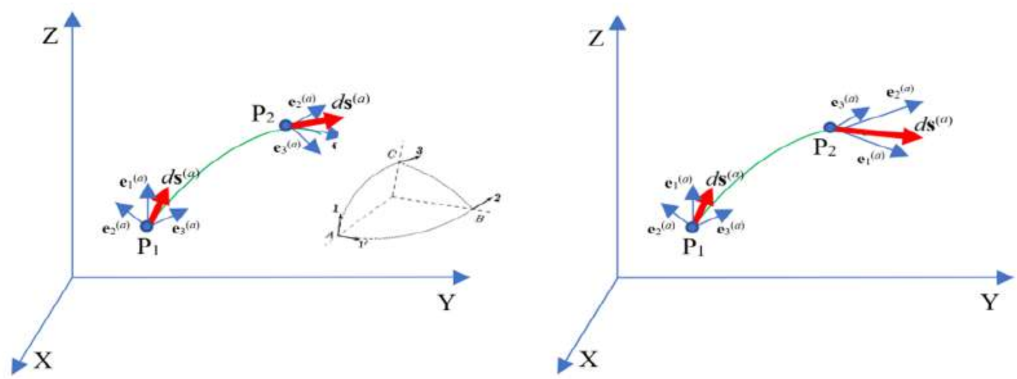
This work is the fifth in a series of articles under the general title "Geometrized vacuum physics based on the Algebra of Signatures." The purpose of this project is to study the properties of emptiness (i.e. "vacuum"). In this regard, in the previous four articles of this series (Batanov-Gaukhman, 2023a; Batanov-Gaukhman, 2023b; Batanov-Gaukhman, 2023c; Batanov-Gaukhman, 2023d), a method was proposed for deep probing of the "vacuum" by illuminating it with mutually perpendicular monochromatic rays of light with wavelengths $\lambda_{m,n}$ from all wave subranges $\Delta\lambda = 10^m - 10^n$ cm, where $n = m + 1$ (see §§ 1 – 2 in (Batanov-Gaukhman, 2023a)).

As a result, the deep probing method made it possible to represent emptiness (i.e., "vacuum") as an infinite sequence of $\lambda_{m,n}$-vacuum nested within each other (i.e., light $3D_{m,n}$-landscapes, see Figures 2 and 4 in (Batanov-Gaukhman, 2023a). Based on this method, a mathematical apparatus was developed under the general name "Algebra of Signatur" (abbreviated "Alsigna"), suitable for describing the properties not only of "vacuum", but of any other continuous medium, if these media are probed not with light rays, but, for example, rays of sound waves.

In particular, in (Batanov-Gaukhman, 2023a; Batanov-Gaukhman, 2023b; Batanov-Gaukhman, 2023c; Batanov-Gaukhman, 2023d) the following were stated:
- basics of the Algebra of Stignatures (for a set of 4-dimensional affine, i.e. vector, spaces);
- basics of the Algebra of Signatures (for a set of 4-dimensional metric spaces);
- basics of spectral-signature analysis;
- some aspects of kinematics and dynamics of $\lambda_{m,n}$-vacuum layers.

Each of these areas of research requires further development, but this article takes the next step in the direction of developing Alsigna's mathematical apparatus, in particular, the possibility of a geometrized description of stable vacuum formations is considered. These are, such curved areas of "vacuum" that do not change over time.

In this article we will use the simplest version of differential geometry, with simplifications corresponding to Riemannian geometry (see Figure 1*a* or Figure 4 in (Batanov-Gaukhman, 2023d)). We will call this type of simplification the Riemannian approximation.



a) *b*)

Fig. 1: (repetition of Fig. 4 in (Batanov-Gaukhman, 2023d)) *a*) In Riemannian geometry, the transferring of the vector $ds^{(a)}$ from point p_1 of a curved space to the nearby point p_2 of the same space is carried out along a tangent to the geodesic line connecting these points. In this case, only the direction of this vector changes, and its magnitude remains unchanged. In this case, when transferring the vector $ds^{(a)}$, the magnitude of the basis vectors $e_m^{(a)}$ and the angles between them do not change.

The most complex version of differential geometry corresponds to a distorted space, in which the geodesic line between two nearby points p_1 and p_2 of this space is not only curved, but also twisted, deformed (stretched or compressed) and displaced. In this case, when transferring the vector $d\mathbf{s}^{(a)}$ tangentially to such a geodesic line, it can change: direction, length, displacement, it can rotate along with the twisting of the geodesic line (see Figure 1*b*). We will call this most complex differential geometry "spacemetry of meta-absolute parallelism" (abbreviated as MAP-spacemetry).

MAP-spacemetry has yet to be developed despite the fact that much has already been done in this direction. For example, the following have been developed: Riemann-Cartan-Schouten geometry with torsion, Einstein-Weyl geometry, Weizenbeck-Vitali-Shipov geometry of absolute parallelism, Newman-Penrose isotropic tetrad method, Rosen bimetric geometry, complex Riemannian geometry, Finsler geometry, teleparallel Hornsdesky gravity models, Randall-Sundrum gravity models, loop quantum gravity model, Brans-Dicke gravity model, Gauss-Bonet gravity model, conformal gravity, etc.

As will be shown below, the Riemannian approximation (i.e., geometry with simplifications shown in Figure 1*a*) allows us to obtain metric-dynamic models of stable vacuum formations of the corpuscular type. But to describe stable nodal vacuum structures Riemannian geometry and Algebra ща Signature are not enough.

Let's note once again that the purpose of this article is to construct metric-dynamic models of stable vacuum formations, based on simplified Riemannian geometry and the Algebra of Signatures (Alsigna), presented in the first articles of the proposed project (Batanov-Gaukhman, 2023a; Batanov-Gaukhman, 2023b; Batanov-Gaukhman, 2023c; Batanov-Gaukhman, 2023d).

To build models of stable vacuum formations, it is necessary to first formulate conservation laws. To do this, we will use the general theory of relativity of A. Einstein, which is based on Riemannian geometry. However, general relativity (GR) is not entirely suitable for achieving this goal for a number of reasons listed below.

1] *Analysis of contradictions in general relativity*

The analysis below of the origin of the basic equation of the general theory of relativity does not pretend to be rigorous and is not the result of a scrupulous study of the numerous literatures devoted to this great "monument" of human thought. This is only an attempt to reconstruct the sequence of events in order to identify the root of the contradictions in this theory.

Initially, A. Einstein, over the course of 10 – 12 years (from 1906 to 1917), built the general theory of relativity in such a way that for a non-relativistic approximation (i.e. for a weak gravitational field and low velocities) it was reduced to Newton's theory of gravitation.

In Newtonian mechanics, the potential of the gravitational field φ created by a material body with mass density ρ is described by the Poisson equation

$$\Delta\phi = 4\pi G\rho, \tag{1}$$

where $G = 6.674\cdot10^{-11}\ \mathrm{N\cdot m^2/kg^3}$ – gravitational constant.

Outside a massive body, Poisson's equation (1) turns into Laplace's equation $\Delta\phi = 0$, the solution of which for a spherical body with constant mass M has the form

$$\phi = -G\frac{\mathrm{M}}{r}, \tag{2}$$

where $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$ is the distance from the center of the body to the observation point.

The criterion for the truth of new ideas about the nature of gravity for A. Einstein was the possibility of returning to the Poisson equation (1) while simplifying the initial conditions.

In addition to the condition of continuity of theories, A. Einstein was also guided by: the principle of coordinate invariance (i.e., the independence of the laws of physics from the choice of coordinate system), the principle of general covariance (i.e., the statement that equations describing physical phenomena in different coordinate systems and systems reference systems must have the same form. In particular, the equations must be invariant with respect to Lorentz transformations). A. Einstein also relied on the heuristic principle of "equivalence of the forces of gravity and inertia" (i.e. the force of gravitational interaction was identified with the force of inertia arising in the accelerated frame of reference). In other words, Einstein compared the effects of gravity with the curvature of 4-dimensional space-time. Another important Einsteinian principle is the "independence of the speed of light from the reference frame," which ultimately connected space and time into a single Minkowski space-time continuum with the metric $ds^2 = -c^2dt^2+dx^2+dy^2+dz^2$. At the beginning of his creative career, A. Einstein was inspired by the ideas of E. Mach that the characteristics of space and time (in particular, the properties of inertial reference systems) are predetermined by the distribution of massive bodies. Einstein also agreed with Mach's criticism of Newtonian physics regarding absolute space, absolute motion and absolute mass, from which it followed that all equations of physics should include only relative quantities, for example, relative distances, relative velocities and relative inertia. But subsequently, the long-range action of Newton's gravity (which E. Mach adhered to) came into conflict with the limit of the speed of light, and the conditions for the formation of inertial forces with the principle of equivalence. Therefore, within the framework of Einstein's special and general theories of relativity, Mach's ideas changed beyond recognition.

In the period 1913 – 1915 Albert Einstein, with the assistance of Marcel Grossmann, took advantage of the achievements of Riemannian geometry, generalized to the case of curved 4-dimensional space-time based on the work of Hendrik Lorentz, Henri Poincaré and Hermann Minkowski. Einstein, together with Adrian Fokker, was also influenced by Gunnar Nordström's nonlinear theory of gravity.

As a result, in the middle of 1915, Einstein wrote down the generally covariant equation (Einstein, 1915)

$$\sqrt{-g}R_{ik} = -\varkappa\left(T_{ik} - \frac{1}{2}g_{ik}\mathrm{T}\right), \tag{3}$$

where g_{ik} are the components of the metric tensor of a curved 4-dimensional space with the metric $ds^2 = g_{ik}\,dx^idx^k$;

$g = |g_{ik}|$ is the determinant of the matrix g_{ik}. (4)

$\varkappa$ – proportionality coefficient;

$R_{ik} = \frac{\partial\Gamma^l_{ik}}{\partial x^l} - \frac{\partial\Gamma^l_{il}}{\partial x^k} + \Gamma^l_{ik}\Gamma^m_{lm} - \Gamma^m_{il}\Gamma^l_{mk}$ is Ricci tensor; (5)

$\Gamma^\lambda_{ik} = \frac{1}{2}g^{\lambda\mu}\left(\frac{\partial g_{\mu k}}{\partial x^i} + \frac{\partial g_{i\mu}}{\partial x^k} - \frac{\partial g_{ik}}{\partial x^\mu}\right)$ are Christoffel symbols; (6)

T_{ik} is energy-momentum density tensor of a material object;

T is trace of the energy-momentum density tensor, $T = g^{ik}T_{ik}$.

It is very difficult to understand the incredibly intense thought process of A. Einstein, but, apparently, he equated the fully geometrized Ricci tensor R_{ik} with the material tensor $T_{ik} - \frac{1}{2}g_{ik}\mathrm{T}$ because in curved space the covariant derivatives of all these tensors are equal to zero

$$\nabla_j R_{ik} = 0, \quad \nabla_j(T_{ik} - \tfrac{1}{2}Tg_{ik}) = 0, \quad \nabla_j T_{ik} = 0, \quad \nabla_j g_{ik} = 0, \tag{7}$$

David Hilbert showed the mathematical incorrectness of Eq. (3).

D. Hilbert in 1915 was in close correspondence with A. Einstein and he, apparently, saw Eq. (3) with a trace term on the right side (Logunov *et al.*, 2004). The presence of the trace term $\frac{1}{2}Tg_{ik}$ in Eq. (3) could serve as a guide for Hilbert in his search for the correct solution.

In the 1915 paper (Hilbert, 1979), D. Hilbert calculated the variation of the integral

$$\delta\int R\sqrt{-g}\,d\Omega = \delta\int g^{ik}R_{ik}\sqrt{-g}\,d\Omega, \tag{8}$$

where $R = g^{ik}R_{ik}$ is scalar curvature;
$d\Omega = dtdxdydz$ is element of 4-dimensional volume. (9)

As a result, Hilbert obtained a tensor with a trace term $R_{ik} - \frac{1}{2}Rg_{ik}$, the covariant derivative of which is equal to zero (Hilbert, 1979)

$$\nabla_j(R_{ik} - \frac{1}{2}Rg_{ik}) = 0. \tag{10}$$

Later it turned out that within the framework of Riemannian geometry the second Bianchi identity is proved.

$$\nabla_i R^s_{rjk} + \nabla_j R^s_{rki} + \nabla_k R^s_{rij} = 0. \tag{11}$$

With simple transformations and multiplication by the contravariant tensor g^{ik}, the Bianchi identity (11) is reduced to Ex. (10). This method of obtaining the Einstein tensor $R_{ik} - \frac{1}{2}Rg_{ik}$ is called "royal" because of its prostate. However, according to many researchers, neither Einstein nor Hilbert knew the Bianchi identities at the time of the creation of the basic equation of general relativity. Both geniuses used the calculus of variations.

Some researchers believe that A. Einstein learned about the tensor with a trace term $R_{ik} - \frac{1}{2}Rg_{ik}$ from the work of D. Hilbert (Hilbert, 1979). Therefore, he multiplied both sides of Eq. (3) by g^{ik}

$$g^{ik}R_{ik} = -\varkappa g^{ik}\left(T_{ik} - \frac{1}{2}g_{ik}\mathrm{T}\right),$$

as a result, he got $T = R/\varkappa$, from which the equation easily follows

$$R_{ik} - \frac{1}{2}Rg_{ik} = \varkappa T_{ik}. \tag{12}$$

From the special theory of relativity A. Einstein knew that the energy-momentum density tensor (or stress–energy tensor) can have the form (Landau & Lifshitz, 1971)

$$T_{ik} = (p + \rho c^2)u_iu_k - pg_{ik} + \frac{1}{4\pi}\left(-F_{il}F^l_k + \frac{1}{4}g_{ik}F_{lm}F^{lm}\right), \tag{13}$$

where ρ is the density of matter; $c = \sqrt{u_0u_0}$ – speed of light; u_i – 4-speed of matter movement; p – pressure, F_{il} – electric field.

For dusty stationary and uncharged matter (i.e., at $p = 0$, $u_x = u_y = u_z = 0$ and $F_{il} = 0$), only one component of the energy-momentum tensor (13) is not equal to zero $T_{00} = \rho c^2$ (Landau & Lifshitz, 1971).

Therefore, at low speeds compared to the speed of light and in the approximation of a weak gravitational field, i.e. $g_{00} \approx 1 + 2\phi/c^2$, Eq. (12) reduces to Poisson's equation (1) if the proportionality coefficient is

$\varkappa = 8\pi G/c^4 \approx 2{,}07665\cdot 10^{-43}\ \mathrm{N}^{-1}$.

In fact, a methodological substitution occurred in this task. It is clear that Einstein was solving a colossally complex problem, and it was important for him that in the non-relativistic (Newtonian) limit, Eq. (12) was reduced to the Poisson equation (1). But this happened due to intricate manipulation of relativistic mass. As a result, a fitting parameter arose, the famous $E = mc^2$, *which was substituted into the classical non-relativistic Lagrangian*

$$L = -mc^2 + \frac{mv^2}{2} - m\varphi.$$

The constant value mc^2 *in the Lagrangian does not affect the equation of motion of a material object, but if this enormous energy of a body at rest (included in the consideration beyond any common sense) is removed from this Lagrangian, then the Poisson equation (1) from the Einstein-Hilbert equations (12) at low speeds and a weak field it will not work. That is, without a purely relativistic correction* mc^2, *no non-relativistic classical limit can be obtained from the general relativity equations – this is a paradox in itself.*

As a result, this adjustment led to an incorrect result. If the solution (2) $\phi = -GM/r$ *) to substitute into the metric with a zero component* $g_{00} \approx 1 + 2\phi/c^2$

$$ds^2 \approx \left(1 + \frac{2\phi}{c^2}\right)c^2dt^2 - d\vec{r}^2 \approx \left(1 - \frac{2GM}{c^2}\frac{1}{r}\right)c^2dt^2 - d\vec{r}^2$$

then this metric will not be a solution to Eq. (12) with $T_{00} = \rho c^2$ *and* $M = \int \rho dV$

$$R_{00} - \frac{1}{2}Rg_{00} = \frac{8\pi G}{c^4}\rho c^2.$$

In the best case, this metric is the Schwarzschild solution of the vacuum equation $R_{ik} = 0$.

"Nobody understands quantum mechanics," – said Richard Feynman, and no one understands the theory of relativity. Historians of science say that after the lecture, enthusiastic students said to Arthur Eddington: "You are the second person in the world who understands the general relativity!" Eddington responded by asking: –"Who's first?"

Can this be considered Einstein's mistake? Of course not. Firstly, Einstein was sincere in his calculations, because the result obtained convincingly followed from the special theory of relativity. Secondly, he completely repeated the logic of post-Newtonian physics, because potential (2) was a solution to the equation $\Delta\phi = 0$. *Third, this misconception was historically inevitable. At that time, the authority of classical physics was so indisputable that if Newton's theory of gravity had not followed from GTR in the non-relativistic limit, the new theory would not have been accepted.*

Thus, the coefficient $8\pi G/c^4$ on the right side of Eq. (12) was introduced by A. Einstein in order to harmonize the dimensions of the two sides of this equation, and so that, under the condition of a weak gravitational field, the Poisson equation (1) would follow from Eq. (12).

As a result, by the end of 1915, A. Einstein and D. Hilbert almost simultaneously obtained a general covariant equation connecting the metric characteristics of a local region of curved 4-dimensional space with the components of the stress–energy tensor of matter

$$R_{ik} - \frac{1}{2} R g_{ik} = \frac{8\pi G}{c^4} T_{ik}. \tag{14}$$

It is forced, by connecting the right side of Eq. (14) with the phenomenological properties of unknown matter (terra incognita), A. Einstein introduced several problems into general relativity.

The first problem of general relativity is due to the presence on the right side of Eq. (14) of the substance mass density ρ with a voluntaristic dimension kg/m^3 and with a dimensional constant G (N·m^2/kg^3), which in principle cannot be introduced into a fully geometrized theory.

Let us recall that kilogram (kg) is a subjective, phenomenological concept. Until May 20, 2019, one kilogram in the SI system was understood as the "mass" of a platinum-iridium cylinder with a diameter and height of 39.17 mm (i.e., the international prototype of the kilogram), the weight of which corresponds to the weight of a cubic decimeter (liter) of distilled water at a temperature of 4 °C and an atmospheric pressure of 101.325 kPa at the latitude of Paris. It is obvious that the kilogram dimension is a purely voluntaristic concept and is in no way related to geometry.

The gravitational constant is an extremely small value $G = 6.67430(15)\cdot 10^{-11}$ m^3·s^{-2} kg^{-1}, which is determined from the average mass density of the Earth. with a large relative error of ~10^{-4}, which has not been reduced for many decades. At the same time, the very density of the mass of our planet is determined by indirect (far from obvious) methods. There is also no certainty that the gravitational constant G is the same throughout the Universe, and that it does not change over time.

An attempt to substantiate the value of the gravitational constant G was made in the Jordan-Brans-Dicke theory of gravitation by introducing a scalar potential φ interacting with the space-time metric. However, within the framework of this theory, G is not necessarily constant, but depends on the scalar field 1/G˜φ, which can vary in space and time. Despite the fact that this theory of gravity reduces to general relativity in the limiting case, a number of its predictions have not been confirmed in practice. In addition, this theory has an additional adjustable coupling parameter ω, which entails replacing one empirical constant with another.

The second problem of GR is related to the possibility of violation of the nonlocal laws of conservation. The point is that conservation laws must have the form (Landau & Lifshitz, 1971)

$$\frac{\partial T_{ik}}{\partial x^j} = 0, \tag{15}$$

whereas in a curved space the covariant derivative is equal to zero

$$\nabla_j T_{ik} = \frac{\partial T_{ik}}{\partial x^j} - \Gamma_{ij}^l T_{lk} - \Gamma_{kj}^l T_{il} = 0, \tag{16}$$

which differs from the conservation law (15) by the amount $-(\Gamma_{ij}^l T_{lk} + \Gamma_{kj}^l T_{il})$.

Indeed, the integral over a 4-dimensional volume $\int T_{ik}\sqrt{-g}\, d\Omega$ is preserved only if the satisfied condition (Landau & Lifshitz, 1971)

$$\frac{\partial \sqrt{-g} T_{ik}}{\partial x^j} = 0. \tag{17}$$

Only for a locally inertial reference frame in which all Christoffel symbols are equal to zero ($\Gamma_{kj}^l = 0$), a full-fledged conservation law is obtained $\nabla_j T_{ik} = \partial T_{ik}/\partial x^j = 0$.

GR apologists associated the violation of nonlocal conservation laws with the fact that the Einstein-Hilbert equation (14) is not complete, because it does not include the energy-momentum of the gravitational field t_{ik} itself, defined by such a pseudo-tensor that:

$$\frac{\partial}{\partial x^j}(-g)(T_{ik}+t_{ik})=0. \tag{18}$$

One of the explicit types of pseudo-tensor t_{ik} is written in (Landau & Lifshitz, 1971):

$$t^{ik}=\frac{c^4}{16\pi G}\{(2\Gamma^n_{lm}\Gamma^p_{np}-\Gamma^n_{lp}\Gamma^p_{mn}-\Gamma^n_{ln}\Gamma^p_{mp})(g^{il}g^{km}-g^{ik}g^{lm})+g^{il}g^{mn}(\Gamma^k_{lp}\Gamma^p_{mn}-\Gamma^k_{mn}\Gamma^p_{lp}-\Gamma^k_{np}\Gamma^p_{lm}-\Gamma^k_{lm}\Gamma^p_{np})+ \\ +g^{kl}g^{mn}(\Gamma^i_{lp}\Gamma^p_{mn}-\Gamma^i_{mn}\Gamma^p_{lp}-\Gamma^i_{np}\Gamma^p_{lm}-\Gamma^i_{lm}\Gamma^p_{np})+g^{lm}g^{np}(\Gamma^i_{ln}\Gamma^k_{mp}-\Gamma^i_{lm}\Gamma^k_{np})\}. \tag{19}$$

However, if the pseudo-tensor t_{ik} were included in the right side of Eq. (14), then, according to the logic of general relativity, this would mean that the curvature of space would be the source of its own curvature with infinitely complex consequences. In addition, it turned out that all types of pseudo-tensors t_{ik} are associated with problems such as the "Bauer paradox" (Vladimirov, 2005), because all known pseudo-tensors t_{ik} turn out to be non-zero even for a flat pseudo-Euclidean space, the metric of which is given in curvilinear coordinates.

The problem of violation of the law of conservation of energy in general relativity is also present in another capacity. When a body falls into a black hole, its energy tends to infinity even when approaching the gravitational radius.

A. Einstein realized that the right side of Eq. (14) is phenomenological in nature. There is an opinion in scientific circles that Einstein called the left side of this equation a "Magnificent Palace" and the right side a "ramshackle hut." Einstein himself and many of his followers repeatedly tried to geometrize the right-hand side of Eq. (14) by complicating the properties of space-time, considering, for example, space-time with torsion, or space with five (the theories of Kaluza and Klein) or more dimensions.

All these works are associated with the program of "complete geometrization of physics" by William Clifford (Clifford, 1870). A review of various attempts to geometrize the right-hand side of the Einstein-Hilbert equation (14) can be found, for example, in (Alekseev, 2021).

However, many varieties of geometric-physics face other kinds of difficulties. For example, in non-Riemannian geometries, torsion and nonholonomic objects cannot be the reason for the long-term existence of stable vacuum formations, because torsion and local spin-torsion manifestations can only describe rotating (vortex-like) regions of vacuum that are soliton in nature, i.e. existing only as long as they move at a speed consistent with the "elastic-plastic" properties of vacuum.

The third problem of GR is the following. As noted by V.V. Karbanovsky, due to the symmetry of the tensors $R_{ik}=R_{ki}$, $g_{ik}=g_{ki}$, $T_{ik}=T_{ki}$ the Einstein-Hilbert differential equations (14) are reduced to a system of ten equations, but variable parameters (i.e. unknown quantities) in there are twenty of these equations

$$\begin{pmatrix} g_{00} & g_{10} & g_{20} & g_{30} \\ g_{01} & g_{11} & g_{21} & g_{31} \\ g_{02} & g_{12} & g_{22} & g_{32} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{pmatrix} \text{ и } \begin{pmatrix} T_{00} & T_{10} & T_{20} & T_{30} \\ T_{01} & T_{11} & T_{21} & T_{31} \\ T_{02} & T_{12} & T_{22} & T_{32} \\ g_{03} & T_{13} & T_{23} & T_{33} \end{pmatrix}. \tag{20}$$

Therefore, it is almost impossible to solve these equations without additional conditions and irremovable uncertainties.

For example, let us consider the Friedmann-Lemaitre-Robertson-Walker metric (FLRW-metric), which is largely fundamental in modern astrophysics.

$$ds^2 = -c^2dt^2 + a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right), \tag{21}$$

where $k = 0, 1, -1$;

$a(t)$ – "scale factor", intended for transition to the accompanying reference frame, depends on time *t*, which flows equally at all points of a homogeneous and isotropic universe, which has the properties of an "ideal fluid" with the same average mass density ρ and pressure p everywhere.

The stress-energy tensor at each point of such an "ideal fluid":

$$T_i^k = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}. \tag{22}$$

The FLRW-metric (21) is not a solution to the Einstein-Hilbert equation (14) in the classical sense of the word "solution". In fact, this metric is first constructed from the assumption that each local region of 4-dimensional space is a 3-pseudosphere with a radius $a(t)$ depending on time. The equation of such a local 3-pseudosphere has the form

$$-dx_2^2 + dx_1^2 + dx_2^2 + dx_3^2 = ka(t)^2, \tag{23}$$

where $k = 0, 1, -1$.

Mathematical transformations of the 3-pseudosphere equation (23) lead to metric (21).

Next, in order to find out how the volume of each 3-pseudosphere with radius a(t) can change within the framework of general relativity, the components of the metric (21) are substituted into the Christoffel symbols Γ_{ik}^{λ} (6). In turn, the values of the calculated symbols Γ_{ik}^{λ} are substituted into the Ricci tensor (5), and the resulting components of the Ritchie tensor are substituted into the Einstein-Hilbert equation (14). The result is a system of Friedmann equations

$$\begin{cases} \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho, \\ 2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = -\frac{8\pi G}{c^2}p, \\ \rho = f(p), \end{cases} \tag{24}$$

where $\dot{a}, \ddot{a}$ – are the first and second derivatives of the scale factor $a(t)$;

Eq. (14), and the Friedman equations (24) following from it, did not allow the possibility of describing a stationary Universe. Therefore, A. Einstein in 1917 took advantage of the property of covariant derivatives (7), in particular $\nabla_j g_{ik} = 0$, and in article (Einstein, 1917) he wrote down an expression with the lambda term Λ, which transforms into the formula

where Λ is a constant called the "cosmological constant".

When substituting the components of the metric tensor from the FLRW-metric (21) into Eq. (25), we obtain a system of Friedman equations with a lambda term

$$\begin{cases} \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}\rho, \\ 2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \Lambda c^2 = -\frac{8\pi G}{c^2}p, \\ \rho = f(p). \end{cases} \quad (26)$$

Since systems (24) or (26) have two equations, and in the first cas e there are three unknowns: $a(t), \rho$ and p in the second case there are four unknowns $(t), \rho, p$ and Λ. Therefore, it is necessary to add additional equations, such as $\rho = f(p)$ or $p = f(\rho, \Lambda)$, which are called "state equations". There can be an infinite number of such equations of state, entered "by hand". It should also be taken into account that k can take any of three values 0, 1, −1. In addition, when solving differential Eqs. (24), integration constants arise, which are also eliminated voluntarily, because the boundary conditions in these problems are often undefined.

The main problem, however, is that solutions to the Friedmann equations (26) in the presence of an additional equation of state $\rho = f(p)$ will not be solutions to the Einstein equation (25). This "hand-introduced" voluntaristic function changes the results of solving differential equations. The Friedmann system of equations (26) and the Einstein equation (25) completely coincide only in one case, if $\rho = 0$ and $p = 0$, i.e. if $\mathrm{T}_{ik} = 0$.

The fourth problem of GR is that the equality sign between the space curvature tensor $G_{ik} = R_{ik} - \frac{1}{2}Rg_{ik}$ and the stress-energy tensor of matter T_{ik} suggests the possibility of formulating as a direct problem (i.e. determining the curved state of the space-time continuum with a known distribution and movement of matter), and the inverse problem (i.e., determining the distribution and movement of matter with a known curvature of the space-time continuum). Such a closed interdependence in the case of strong curvature of 4-space and high energy density of matter leads to insoluble uncertainties and contradictions. In other words, Eq. (14) and Eq. (25) are partly suitable only for the case of weak gravity and low energy density of matter.

The 1983 Nobel Prize winner Subrahmanyan Chandrasekhar in (Chandrasekhar, 1983) writes not about trust, but about faith in GR: "For the last twenty years, great efforts have been aimed at testing the lower orders of approximation of general relativity and Newtonian theory. These efforts were crowned with success, and the predictions of general relativity related to the change in the flow of time at points with different gravity, to the deflection of light rays expected when crossing the gravitational field and to the precession of Keplerian orbits and, finally, to the slowing down of the orbital period of binary stars in eccentric orbits, due to gravitational radiation, everything was confirmed within the limits of observation and measurement errors. But all these differences of GR in relation to the consequences of Newtonian physics amounted to several parts in a million. However, within the limits of a strong gravitational field, general relativity has not yet received unambiguous confirmation... Why then do we believe this theory? … our trust follows from the beauty of the mathematical description of nature that GR provides." Classical mechanics earned the trust of scientists and engineers after the work of Newton, Euler, and Laplace, showing its effectiveness in a variety of applications. In relation to GR, Chandrasekhar speaks only about faith (Burlankov, 2011). L. Brillouin in (Brillouin, 1972) also expressed solidarity with this opinion: "The general theory of relativity is an example of a magnificent mathematical theory built on sand."

The fifth problem of GR is the amazing uselessness of the Einstein-Hilbert equation (14) with the right-hand side not equal to zero ($\mathrm{T}_{ik} \neq 0$) and with the dimensional proportionality coefficient $\varkappa$. This greatest crown of human thought turned out to be practically useless.

Firstly, the possibility of applying Eq. (14) is strongly limited by the extremely small value of the Einstein constant $\varkappa = 8\pi G/c^4 \approx 2{,}07665\cdot10^{-43}\ \mathrm{s^2/(kg\cdot m)} = \mathrm{N^{-1}}$, since in this case the curvature of space begins barely manifest in the presence of enormous energy densities.

Secondly, only the vacuum equations $R_{ik} = 0$ and $R_{ik} + \Lambda g_{ik} = 0$ for $\mathrm{T}_{ik} = 0$ can be strictly solved. The presence of matter parameters in solutions of vacuum equations is always ephemeral, i.e. they are introduced by "hands" in the form of fitting parameters or material equations of a phenomenological nature. This is the case when determining the additional perihelion shift of Mercury's orbit, and when estimating the deflection of a ray of light in the gravitational field of the Sun and when solving Friedman's equations. In the first two cases, the Schwarzschild metric is used (i.e. solution of the vacuum equation), describing the curvature of an empty space-time continuum, and the mass of the Sun is inserted into this metric "manually" as a correction factor.

On the contrary, Einstein's vacuum equations do not have dimensional constants on the right side, so they find many applications in various branches of knowledge. For example, A. Einstein himself and his student Nathan Rosen in 1935 proposed to consider an electron as a merger of two pico-scopic "black holes", which are described by stitching together two Schwarzschild metrics. This idea turned out to be untenable, but the Einstein-Rosen "bridges" (i.e., "wormholes") still remain the focus of attention of scientists, because they open up the possibility of interstellar and intergalactic travel, as well as time travel, as suggested by the groups of Kip Thorne and Igor Novikov (Alekseev, 2021)

In addition, "strong gravity" was developed in the works of several theorists, including Abdus Salam and Erasmo Recami (Salam & Strathdee, 1978; Recami & Castorina, 1976; Oldershaw, 2006; Fedosin, 2012; Pavsic, 1978). This line of research emerged in the 1960s as an alternative to quantum chromodynamics (QCD). The hypothesis of the existence of "strong gravity" led to an attempt to explain the problem of quark confinement using the "hadron bag" model (i.e., the de Sitter microverse). In this case, the hadron radius was determined by the micro-cosmological constant.

Also, under the assumption of the presence of strong gravitational interaction, analogies between hadrons and black holes of the Kerr-Newman type are described. This approach also did not lead to positive results, but in string theory there is a close connection between gauge forces and the geometry of spacetime. In some cases, string theorists recognize important analogies between theories based on Einstein's general theory of relativity and Yang-Mills gauge theory (in particular, quantum chromodynamics (QCD) and the theory of electroweak interactions by S. Glashow, S. Weinber and A. Salam) (Burlankov, 2011).

There are studies that show that with simplifications corresponding to Riemann geometry, the nonlinear equations of the Yang-Mills theory are reduced to the form of Einstein's vacuum equations (Krivonosov & Lukyanov, 2009).

Other applications of Einstein's vacuum equations are the description of gravitational lenses and gravitational waves, for which the LIGO and VIRGO collaborations were awarded the Nobel Prize in Physics in 2017.

The sixth problem of GTR is the presence of singularities (i.e., tendencies to infinity) in solutions to the Einstein-Hilbert equation (14) (more precisely, in solutions to Friedmann equations (24) and (26)). The same problem remains in solutions of Einstein's vacuum equations. When the Schwarzschild metric was published, the scientific community (starting, apparently, with a discussion in the "Collège de France", which took place in 1922 with Einstein, Hadamard, Painlevé, Becquerel, Brillouin, Cartan, Langevin and other scientists) were very worried about the presence of a singularity in it. Many attempts have been made to get rid of this problem by moving to other reference systems, but without success. Gradually they got used to singularities, or rather, some of them were hidden in the distant past, some in the distant future, and the rest were drowned in the bottomless depths of black holes, hiding behind the "principle of cosmic censorship" of Roger Penrose (Penrose, 1973). Nevertheless, the problem remained along with the understanding that the presence of singularities in any theory is a clear indicator of its incompleteness.

The seventh problem is related to time loops. Einstein's collaborator at the Institute for Advanced Study, Kurt Gödel in (Gödel, 1949) obtained an exact solution to Eq. (25), allowing the existence of closed time-like lines. This solution is generated by the stress-energy tensor T_{ik}, which is the matter density of uniformly distributed rotating dust particles. Gödel's solution is expressed as a metric tensor in the local coordinate system

$$ds^2 = \frac{1}{2\omega^2}\left(-(cdt + e^x dz)^2 + dx^2 + dy^2 + \frac{1}{2}e^{2x}dz^2\right),$$

where $-\infty < t, x, y, z < \infty$;
ω is a non-zero real constant representing the angular velocity.

In this case, the principle of causality is violated. If a closed time-like line returns to the same point from which the movement was started, then it describes an arrival at the same "time" that has already "been." Moreover, for the researcher who observed this line, time is not zero. Thus, we get a closed chain of causes and effects along this line.

Einstein was alarmed by the presence of this Gödel solution, he noted (Einstein, 1966): – "It would be interesting to find out whether such decisions should perhaps be excluded from consideration on the basis of physical considerations." However, this solution of Eq. (25) in itself with a non-zero the stress-energy tensor ($T_{ik} \neq 0$), leading to a cosmological model of a rotating Universe, does not cause rejection. The problem is the many paradoxes associated with the possibility of time travel. We now know about Edward Lorenz's "butterfly effect" and understand that the slightest change in the past can completely change the future. Hence the hypothesis about the security of chronology, proposed by Stephen Hawking. Let's note, however, that mental (i.e., disembodied, purely observational) presences in the past and future are not prohibited. Paradoxes are associated with the transfer into the past of a material body that can change the course of history by a small impact. In other words, if traveling to the future or the past is still possible, then most likely without the transfer of matter there, i.e. at $T_{ik} = 0$.

The eighth problem of GR is related to the illusory nature of matter. V. Karbanovsky, referring to Taimuraz Kairov, noted that within the framework of Riemannian geometry, by choosing the gauge function h^{mkl} one can always reset to zero the curvature tensor in a local region of curved space (Karbanovsky, 1992)

$$R^i_{mkl}h^{mkl} = 0.$$

That is, in any local region of curved space it is always possible to switch to a tangent coordinate system and ensure that the Riemann-Christoffel curvature tensor R^i_{mkl}, and therefore the Ritchie tensor R_{ik}, are equal to zero in this small region.

Let's recall that the main theorem of Riemannian geometry says: "By definition, every Riemannian space in the infinitesimal coincides with Euclidean space up to small 1st order (with respect to differentials) coordinates." It turned out that between the Riemannian space *R* and the Euclidean space tangent to it in the neighborhood U_A of some point *A* it is possible to establish such a correspondence in which both spaces will coincide up to small ones above the 2nd order. To do this, in Riemannian space, geodesics are drawn from point *A* in all directions and each of them in the tangent space E_A is compared with a ray of the corresponding direction, and then a correspondence between these geodesics and rays is established such that the lengths of the arcs of the geodesics and the corresponding rays are equal. In a sufficiently small area of point A, such a correspondence will be one-to-one, and this is what we are looking for. Namely: if we introduce Cartesian coordinates $x_1,..., x_n$ in the tangent space and assign their values to the corresponding points of the neighborhood U_A, then the following connection will take place between the linear elements ds of the Riemannian and ds0 of the Euclidean spaces:

$$ds^2 - ds_0^2 = ds_0^2 - \frac{1}{3}\sum_{mlki} R_{mlki}\,(x^m - x_A^m)(x^k - x_A^k)dx^l dx^i + \sum_{mlki} \varepsilon_{mlki}\,(x^m - x_A^m)(x^k - x_A^k)dx^l dx^i,$$

where $\varepsilon_{mlki} \to 0$ for $x^i \to x^i_A$, $i = 1,2,3\ldots n$;

$$R_{mlki} = \sum_s g_{is} \left(\frac{d\Gamma^s_{km}}{dx^l} - \frac{d\Gamma^s_{kl}}{dx^m}\right) + \sum_p \left(\Gamma^s_{lp}\Gamma^p_{km} - \Gamma^s_{mp}\Gamma^p_{kl}\right)$$

is the Riemann-Christoffel tensor, which characterizes the difference between Riemannian space and Euclidean space.

Local zeroing of the Riemann-Christoffel curvature tensor according to the logic of general relativity means that there is no matter in this region, and cannot be, because the equations are tensor. Thus, by choosing the gauge function h^{mkl} (or selecting a local coordinate system), it is possible to achieve that in the neighborhood of a small region, Eq. (14) is reduced to an equation of the form

$0 = \frac{8\pi G}{c^2} T_{ik}$ or $\mathrm{T}_{ik} = 0$.

Thus, the logic of general relativity allows for local zeroing of the energy density (or matter density). At the same time, the tensor nature of Eq. (14) suggests that if matter is illusory in one local reference system, then it must be illusory in all other reference systems associated with a given local region.

In a dynamic system on extremals, the increment of action is determined by the expression $dS = Hdt$, however, if the action does not change with time variation, the Hamiltonian *H* must be equal to zero (Burlankov, 2011). The equality to zero of the Hamiltonian manifests itself in any system that is invariant under a change of time variable. In particular, in the general theory of relativity, the principle of general covariance allows any transformation of variables, including time, therefore the energy of any system in general relativity is exactly zero (Burlankov, 2011). Not only energy as a whole, integrally, but also energy density at any point and at any moment in time. This phenomenon is described in detail in the monograph by Misner, Thorne & Wheeler (Misner *et al.*, 1977): $H(g_{ik}) = 0$, i.e. $E = 0$.

Since in classical physics time is completely defined, energy in the general case is not equal to zero, then general relativity with a non-zero material right-hand side cannot, in any limit, pass into classical physics. In classical physics, the development of the World takes place in global time and any generalization of classical mechanics must contain this time (Burlankov, 2011).

In geometrodynamics, the expanding cosmological model is associated with the density of matter only as a parameter, and in general there are solutions without matter (i.e. $\mathrm{T}_{ik} = 0$). The rigid link between the density of matter and the rate of expansion, described by the Hubble parameter, is the result of the requirement that the total energy of matter and the energy of dynamic space be equal to zero. This condition, as is known, is not satisfied 5–25 times. It was to eliminate this enormous contradiction between observations and predictions of general relativity that "dark energy" was introduced into consideration (Burlankov, 2011).

The ninth problem of "global time" is closely related to the above. The violation of general covariance with the introduction of global time associated with any matter (for example, with the ether) is due to the selection of a global reference system. The solution to this problem faced the need to select the mechanical properties of the ether so that the laws of interaction of bodies and the electromagnetic field with the ether do not depend on the speed of their movement. It turned out that the state of rest of the ether cannot be observed (Burlankov, 2011). Within the framework of general relativity, at each point of the curved space-time continuum, local (proper, or true) time flows in its own way depending on the zero components of the metric tensor $d\tau = c^{-1}\sqrt{g_{00}}\, dt$. Related to this are problems of synchronizing processes in various regions of curved space, as well as problems with defining the concept of energy in general relativity, since energy, from the point of view of mathematical physics, is a quantity that is conserved due to the homogeneity of time.

At the same time, it is obvious that the consistency of many natural phenomena is subject to the flow of a single global time, as is the case in classical post-Newtonian physics and in the ΛCDM standard cosmological model, in which the same global time flows throughout the Universe. However, there is no generally accepted answer to the question: "How does local time in the gravitational field of stars and planets agree with the universal time of the ΛCDM model?" Cosmologists say that galaxies, along with stars, are frozen into space that expands over time. But there is no answer to the question: "How are the gravitational fields of planets, stars and galaxies with their local times linked with the expanding interstellar space with global time? This is a problem, since metrics can only be correctly stitched together with synchronous time at the point of their contact. Despite the fact that many attempts have been made to construct a theory of gravity with violation of general covariance through the introduction of global time, this problem has not been solved to date.

The geometrodynamics of Wheeler, Arnovitt, Deser and Misner (Misner *et al.*, 1977; Arnovitt *et al.*, 1959), without additional conditions for time and without the condition that the Hamiltonian is equal to zero, encountered difficulties in that the fundamental static solutions of general relativity: the Schwarzschild metric, the Reissner-Nordström metric, the Kerr metric turned out to be not representable in dynamic form relative to global space-time (Burlankov, 2011).

The tenth problem of GR is related to the quantization of the gravitational field. Due to general covariance, the Hamiltonian in general relativity is equal to zero, so quantization turned out to be impossible (Burlankov, 2011). An attempt to construct a quantum theory of gravity with a zero Hamiltonian led to the development of the theory of loop quantum gravity (LQG theory). This theory postulates that the structure of space-time consists of finite loops woven into an extremely thin fabric held together by various node connections, which is called a spin network. It is assumed that the cell size of the spin network is of the order of the Planck length

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \approx 1{,}6162 \times 10^{-33} cm.$$

One of the key parameters of loop quantum gravity is the quantized area operator *A* of a two-dimensional surface Σ, which has a discrete spectrum. Every spin network is an eigenstate of each such operator, and the area eigenvalue equals

$$A_\Sigma = 8\pi\, l_p^2 \gamma \sum_i \sqrt{j_i(j_i+1)},$$

where all intersections *i* of the surface Σ with the spin network are summed up. In this formula
γ – Immirzi parameter;
j_i = 0, 1/2, 1, 3/2, ... is the spin associated with the link *i* of the spin network. The two-dimensional area is therefore "concentrated" in the intersections with the spin network.

According to this formula, the smallest possible non-zero eigenvalue of the area operator corresponds to the link that carries the representation with spin 1/2. Assuming an Immirzi parameter of order 1, this gives the smallest measurable area of ~10^{-66} cm^2.

The main role in quantum gravity is played by the uncertainty principle $\Delta r_g \Delta r \geq l_p^2$ (where r_g is the gravitational radius, *r* is the radial coordinate). From this principle it follows $r_g \approx l_p^2/r$. Let's substitute r_g into the Schwarzschild metric (i.e. into the solution of Einstein's vacuum equation $R_{ik} = 0$), as a result we obtain

$$ds^2 = \left(1 - \frac{l_p^2}{r^2}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{l_p^2}{r^2}\right)} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2).$$

This shows that on the scale $r \approx l_p \approx 10^{-33} cm$ black holes should appear, i.e. spacetime must generate quantum foam from real and virtual black holes.

Of course, we are trying here to explain the essence of loop quantum gravity at a primitive level. In reality, theorists are attempting to peer into the deep structure of the void by representing the null Hamiltonian of the vacuum (H = 0) through the introduction of Asteker variables and combinations of Lagrangian factors (i.e., conserved displaced connections) with the $SU(2)$ gauge symmetry group, together with its closed an algebra that transforms into an algebra of Poisson brackets, from which follows a closed algebra of quantum operators. The result is a quantum model of empty space with deeply hidden divergences. All this complex mathematics complements the standard cosmological model, based on the Friedmann equations, only on the scale of Planck lengths ($\sim 10^{-33}$ cm^2) and times ($\sim 10^{-44}$s), which are characteristic of the beginning of the Big Bang, or in the black hole singularity zone.

However, modern technologies make it possible to experimentally test the dimensions of space at least $10^{-16} - 10^{-18}$ cm. Therefore, today it is not possible to verify the theoretical predictions of loop quantum gravity. Despite the fact that a large number of research groups around the world are developing this theory, they have not yet been able to come close to practically significant results.

Even the attempt to quantize Newton's classical theory of gravity encounters numerous difficulties. Quantum gravity turns out to be a non-renormalizable theory due to the fact that the gravitational constant is a dimensional quantity. In the system of units $\hbar = c = 1$, the gravitational constant G has the dimension of the inverse square of the mass. The situation is aggravated by the fact that direct experiments in the field of quantum gravity, due to the weakness of the gravitational interactions themselves, are not available to modern technologies.

We note that the Algebra of Signatures described in (Batanov-Gaukhman, 2023a; Batanov-Gaukhman, 2023b; Batanov-Gaukhman, 2023c; Batanov-Gaukhman, 2023d) largely coincides with the mathematical basis of the theory of loop quantum gravity and the theory of superstrings, but without restrictions on the size of a section of space and the scale of the objects under study.

The eleventh problem is that the GR claims that in the Newtonian limit it goes into classical physics, that is, the principles of general relativity should also operate in post-Newtonian mechanics, but the relativity of time is not observed in the non-relativistic world.

The twelfth, and perhaps the most basic, problem of general relativity is related to the fact that A. Einstein did not explain: "How does the mass of a body, its energy of motion and the pressure inside it bend the space-time continuum?" To the question: "How does the force of gravity arise around a massive body?" Newton replied: "I do not feign hypotheses." Einstein replaced the effect of gravity with free movement by inertia in a curved space-time continuum, but the question of the mechanism for generating this curvature by massive bodies also remained unanswered.

2] *Conclusion of the analysis of general relativity problems*

The difficulties that researchers encounter when solving Einstein's equations for $T_{ik} \neq 0$ are much greater, but the above analysis is enough to draw a general conclusion. Almost all problems of general relativity are related to the phenomenological right-hand side of Eqs. (14) and (25).

In this regard, in this work we will use only the Einstein vacuum equation

$$R_{ik} - \frac{1}{2} R g_{ik} - \Lambda g_{ik} = 0, \quad (27)$$

where Λ can take the values $+\Lambda$, $-\Lambda$ and $\Lambda = 0$. (28)

3] *Massless geometrophysics*

It is necessary not to lose sight of the situation when $T_{ik} = 0$ (i.e. there is no matter), but the Einstein tensor with the lambda term is not equal to zero

$$R_{ik} - \frac{1}{2} R g_{ik} \pm \Lambda g_{ik} = G_{ik} \neq 0. \tag{29}$$

For this matter-free case, we introduce conventional massless notation:

$$G_{ik} = \begin{pmatrix} G_{00} & G_{10} & G_{20} & G_{30} \\ G_{01} & G_{11} & G_{21} & G_{31} \\ G_{02} & G_{12} & G_{22} & G_{32} \\ G_{03} & G_{13} & G_{23} & G_{33} \end{pmatrix} = \begin{pmatrix} W_{tt} & \mathrm{c}S_{xt} & \mathrm{c}S_{yt} & \mathrm{c}S_{zt} \\ 1/_{\mathrm{c}}\, S_{tx} & \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ 1/_{\mathrm{c}}\, S_{ty} & \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ 1/_{\mathrm{c}}\, S_{tz} & \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix}, \tag{30}$$

where

$G_{00} = W_{tt}$ is temporary tension;

$G_{10} = \mathrm{c}S_{10},\ G_{20} = \mathrm{c}S_{20},\ G_{30} = \mathrm{c}S_{30}$ is components of the velocity tension density vector;

$G_{01} = 1/_{\mathrm{c}}\, S_{01},\ G_{02} = 1/_{\mathrm{c}}\, S_{02},\ G_{03} = 1/_{\mathrm{c}}\, S_{03}$ is components of the flux tension density vector;

$G_{\alpha\beta} = \begin{pmatrix} G_{11} & G_{21} & G_{31} \\ G_{12} & G_{22} & G_{32} \\ G_{13} & G_{23} & G_{33} \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{12} & \sigma_{22} & \sigma_{32} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$ is components 3-dimensional spatial tension tensor.

In this case, the massless tensor $G_{ik} \neq 0$ will be called the 4-tension tensor.

Note that the rotational degrees of freedom, particularly the components of the torque density vector, are not taken into account in tensor (30). This is the result of simplifications related to the Riemannian approximation.

Eq. (29) cannot describe stable vacuum formations, since it is impossible to integrate the non-zero tensor field (30) in a curved Riemannian space to ultimately obtain tensor results, because in the general case, similar to Ex. (16)

$$\nabla_j G_{ik} = \frac{\partial G_{ik}}{\partial x^j} - \Gamma^l_{ij} G_{lk} - \Gamma^l_{kj} G_{il} \neq \frac{\partial G_{ik}}{\partial x^j}. \tag{31}$$

Therefore, conservation laws do not work. This means that just by changing the reference system, you can change the energy of the metric-dynamic system and regulate the algorithm for the flow of intra-vacuum processes. For classical physics this sounds categorically unacceptable, but for psychophysics it is a typical phenomenon. For example, if in your mind you mentally form an image of delicious food, then this may be accompanied by real salivation, and a pleasant memory can increase or decrease blood pressure, etc.

Mentally command your hand to rise, and it will rise. Thought is not material, but it forces matter to do work, i.e. brings energy into a material system. The opposite effects are also possible, for example, close your eyes and make 10 revolutions around your axis in a safe place, open your eyes, and you will see that the reference system associated with your consciousness is rotating. These are obvious facts. It is clear that the nervous system transmits a command to contract or relax muscles. But it remains a mystery how the nervous system itself receives a command from thought, which can only form illusory images, i.e. distort local coordinate systems “frozen” into our consciousness?

Equation (29) may be needed when considering how to introduce additional energy into the system by mentally changing the coordinate system and/or reference frame. It is possible that to solve psychomotor problems, Riemannian geometry will not be enough, and it will be necessary to obtain MAP-spacemetry equations (Figures 1*b* and 2*b*). However, when simplified, these equations must still be reduced to the equations of Riemannian geometry.

We demonstrate this using the example of Riemann-Cartan geometry with absolute parallelism. The Riemann-Christoffel curvature tensor in this geometry is identically equal to zero (Ivanenko *et al*., 1985)

$$R^{\beta}_{\beta\mu\nu}(Q) = R^{\beta}_{\beta\mu\nu} + K^{\alpha}_{\beta\nu;\mu} - K^{\alpha}_{\beta\mu;\nu} + K^{\alpha}_{\mu\sigma}K^{\sigma}_{\beta\nu} - K^{\alpha}_{\nu\sigma}K^{\sigma}_{\beta\mu} \equiv 0, \quad (32)$$

where

$R^{\beta}_{\beta\mu\nu}$ is Riemann curvature tensor;

$K_{\mu\nu\lambda} = Q_{\mu\nu\lambda} - Q_{\nu\lambda\mu} + Q_{\lambda\mu\nu}$ is contortion tensor; (33)

$K^{a}_{\mu\nu} = g^{\lambda a}K_{\mu\nu\lambda}$;

$Q^{\lambda}_{\mu\nu} = \frac{1}{2}\left(\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}\right)$ is torsion. (34)

Identity (32) means that in a geometry with absolute parallelism, the components of the Riemannian curvature tensor $R^{\beta}_{\beta\mu\nu}$ turn out to be completely compensated by torsion. Moreover, in this geometry, based on the variational principle, the Einstein-Cartan equation is obtained (Ivanenko *et al*., 1985)

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \pm \Lambda g_{\mu\nu} = Y_{\mu\nu}, \quad (35)$$

where

$Y_{\mu\nu} = K_{\mu}K_{\nu} + K_{\mu\alpha\beta}K^{\alpha\beta}_{\nu} + K_{\alpha\mu\beta}K^{\beta\alpha}_{\nu} + K_{\alpha\beta\mu}K^{\alpha\beta}_{\nu} - \frac{1}{2}g_{\mu\nu}\left(K_{\lambda}K^{\lambda} + K_{\lambda\mu\nu}K^{\lambda\mu\nu}\right)$ is Cartan-Schouten tensor; (36)

$K_{\nu} = 2Q_{\nu} = Q^{\lambda}_{\nu\lambda}$ is the trace of the contortion tensor. (37)

Eq. (35) looks as if the torsion of space, or rather rotational inertia, is the source of its curvature, or, conversely, the curvature of space leads to its torsion.

However, Ex. (31) imposes a restriction on all extensions of Riemannian geometry, including Riemann-Cartan geometry. If $Y_{\mu\nu} = G_{\mu\nu} \neq 0$, then according to (31) this formally means that Eq. (35) cannot serve as conservation laws and cannot describe a stable vacuum formation.

Therefore, to describe stable vacuum formations, the Einstein-Cartan equation (35) must break down into a system of two equations

$$\begin{cases} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \pm \Lambda g_{\mu\nu} = 0, \\ Y_{\mu\nu} = K_{\mu}K_{\nu} + K_{\mu\alpha\beta}K^{\alpha\beta}_{\nu} + K_{\alpha\mu\beta}K^{\beta\alpha}_{\nu} + K_{\alpha\beta\mu}K^{\alpha\beta}_{\nu} - \frac{1}{2}g_{\mu\nu}\left(K_{\lambda}K^{\lambda} + K_{\lambda\mu\nu}K^{\lambda\mu\nu}\right) = 0. \end{cases} \quad (38)$$

This does not contradict Eq. (35), since $G_{\mu\nu} = 0$ and $Y_{\mu\nu} = 0$, therefore $G_{\mu\nu} \equiv Y_{\mu\nu} = 0$.

It is important to note that in the Riemann-Cartan space, due to the asymmetry of the Christoffel symbols $\Gamma^{m}_{ik} \neq \Gamma^{m}_{ki}$, the Ricci tensor also turns out to be asymmetric $R_{\mu\nu} \neq R_{\nu\mu}$. But in the case with $\Lambda = 0$ and $Y_{\mu\nu} = 0$, it follows from Eq. (27) that $R_{\mu\nu} = 0$ and $R_{\nu\mu} = 0$, so they turn out to be identically equal to $R_{\mu\nu} \equiv R_{\nu\mu}$. This corresponds to such types of rotations and torsions of the vacuum that do not affect the Ricci tensor $R_{\mu\nu}$, but they can affect the components of the curvature tensor $R^{\beta}_{\beta\mu\nu}$. It is similar to the fact that a certain volume of space rotates in relation to an external observer, but those who are inside this volume practically do not feel such rotation. For example, being on the surface of the Earth, it is very difficult to feel that it is rotating. However, there are effects that indicate the presence of inertial forces caused by the rotational motion of the planet, for example, deviations of the Foucault pendulum, different steepness of the left and right banks of rivers, etc.

At this stage of the study, we are interested in stable vacuum formations, which are a simplified framework (foundation) for more subtle metric-dynamic effects, therefore it is important to formulate conservation laws within the framework of Riemannian geometry. Einstein's vacuum equation (27) is suitable for this.

Note that in Einstein's vacuum equation (27) there are no problems: neither with mass quantities with the heuristic dimension of kilogram, nor with the dimensional constant G, nor with conservation laws, since substituting $G_{ik} = 0$ into the left side of Ex. (31), we have the coincidence of the covariant and ordinary derivatives

$$\nabla_j 0 = \frac{\partial 0}{\partial x^j} - \Gamma_{ij}^l 0 - \Gamma_{kj}^l 0 = \frac{\partial 0}{\partial x^j} = 0. \quad (39)$$

Einstein wrote (Einstein, 1966): "The gravitational equation for empty space is the only rationally justified case of field theory that can claim rigor."

MATERIALS AND METHOD

1 Vacuum equations and basic ontological principles

1.1 Equation for constructing a metric-dynamic model of a stable vacuum formation

The goal of "Geometrized Vacuum Physics Based on the Algebra of Signature" is the development of one of the main concepts of modern science associated with William Clifford's "Program for the Complete Geometrization of Physics."

Another basis of the Algebra of Signature is the assertion that information is a fundamental concept in physics. According to John Archibald Wheeler's "It from bit" doctrine, all physical entities have an information basis (see (Batanov-Gaukhman, 2023a), in particular §5).

This article is the beginning of an attempt to create a fully geometrized cosmological model without involving the heuristic concept of matter, which has a voluntaristic dimension of the kilogram.

To do this, we first build metric-dynamic models of single stable vacuum formations.

Based on the analysis carried out in the introduction, we use the Einstein vacuum equation (27) for this task

$$R_{ik} - \frac{1}{2} R g_{ik} \pm \Lambda g_{ik} = 0, \quad (27')$$

where

$$R_{ik} = \frac{\partial \Gamma_{ik}^l}{\partial x^l} - \frac{\partial \Gamma_{il}^l}{\partial x^k} + \Gamma_{ik}^l \Gamma_{lm}^m - \Gamma_{il}^m \Gamma_{mk}^l \quad \text{is Ricci tensor;} \quad (5')$$

$$\Gamma_{ik}^\lambda = \frac{1}{2} g^{\lambda\mu} \left(\frac{\partial g_{\mu k}}{\partial x^i} + \frac{\partial g_{i\mu}}{\partial x^k} - \frac{\partial g_{ik}}{\partial x^\mu} \right) \quad \text{is Christoffel symbols.} \quad (6')$$

This equation acts as ten conservation laws.

1.2 Einstein's first vacuum equation

Let's consider Eq. (27) for Λ= 0.

$$R_{ik} - \frac{1}{2} R g_{ik} = 0. \tag{40}$$

Multiplying both sides of this equation by g^{ik}, we obtain (Landau & Lifshitz, 1971)

$$g^{ik}\left(R_{ik} - \frac{1}{2} R g_{ik}\right) = R - \frac{n}{2} R = 0, \tag{41}$$

since $g^{ik} g_{ik} = n$ is the number of dimensions of space.

For any *n*-dimensional space (except *n* = 2), Eq. (41) can only be satisfied for zero scalar curvature (*R* = 0). Therefore, for a 4-dimensional space (i.e. for *n* = 4), Eq. (40) takes a simplified form (Landau & Lifshitz, 1971)

$$R_{ik} = 0. \tag{42}$$

The Ricci tensor (42), which is equal to zero, will be called Einstein's first vacuum equation.

1.3 Einstein's second vacuum equation

If Λ is not zero, then we multiply Eq. (27) by g^{ik}, as a result we obtain

$$g^{ik}\left(R_{ik} - \frac{1}{2} R g_{ik} + \Lambda g_{ik}\right) = R - \frac{n}{2} R \pm n\Lambda = 0, \tag{43}$$

whence follows

$$R = \pm \frac{2n}{n-2} \Lambda, \tag{44}$$

in this case, Eq. (27) takes the form

$$R_{ik} \pm \frac{2}{n-2} \Lambda g_{ik} = 0. \tag{45}$$

In the case of a 4-dimensional space: $n = 4$, $R = 4\Lambda$, and Eq. (45) takes on the simplest (i.e. most optimal) form

$$R_{ik} \pm \Lambda g_{ik} = 0. \tag{46}$$

Eq. (46) will be called Einstein's second vacuum equation.

1.4 Geometric meaning of the constant Λ

Willem de Sitter showed in De Sitter (1979) that a 4-dimensional space can be defined as a conic section of a 5-dimensional single-strip hyperboloid, defined in a 5-dimensional space by the equation

$$x_0^2 - x_1^2 - x_2^2 - x_3^2 - x_4^2 = \pm r_k^2. \tag{47}$$

The curvature tensor of such a 4-dimensional space has the form (De Sitter, 1979; Weil, 2015)

$$R^i_{mab} = \pm\frac{1}{r_k^2}\left(\delta^i_a g_{mb} - \delta^i_b g_{ma}\right). \tag{48}$$

The Ricci tensor in this case is equal to (De Sitter, 1979)

$$R_{im} = R^a_{iam} = \pm\frac{3}{r_k^2}g_{im} \quad \text{or} \quad R_{im} \mp \frac{3}{r_k^2}g_{im} = 0. \tag{49}$$

If you enter the designation

$$\Lambda_k = \pm\frac{3}{r_k^2}, \tag{50}$$

then we get a system of equations

$$\begin{cases} R_{im} + \Lambda_k g_{im} = 0, \\ R_{im} - \Lambda_k g_{im} = 0. \end{cases} \tag{51}$$

which corresponds to Einstein's second vacuum equation (46). But in this case, the geometric meaning of the constant $\Lambda = \pm 3/r_k^2 = \text{const}$ became clear, where r_k is the radius of the 4-dimensional sphere.

Such a 4-sphere has radii along three spatial axes *XYZ* equal to $x_k = y_k = z_k = r_k$, and along the fourth time axis the radius is equal to $ct_k = r_k$.

That is, a given radius is associated with a period of time

$$t_k = r_k / c. \tag{52}$$

The scalar curvature in this case, according to Ex. (44), has the form

$$R = g^{im}R_{im} = 4\Lambda_k = \pm\frac{12}{r_k^2}. \tag{53}$$

The scalar curvature turned out to be proportional to the 12 signs of the Zodiac (i.e., the 12 sectors of the zodiac belt). Zodiac (from the Greek ζῷον – "living being").

1.5 *Epistemological and ontological principles*

Einstein included several important ideas in Eq. (27) in the form of fundamental epistemological principles:
1) The principle of general covariance (i.e., the independence of the form of the equation and invariants from the choice of coordinate system or reference system; in essence, the tensor nature of the equations);
2) The principle of coordinate invariance (i.e., the independence of the laws of physics from the choice of coordinate system);
3) The principle of equivalence (i.e. local curvatures, movements and accelerations are put in correspondence with local reference systems). The concept of "influence of force" has been replaced by inertial movement in curved space-time;
4) The principle of independence of the speed of light from the reference system (i.e., the unification of space and time into a single space-time continuum with a metric of the form $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 = 0$);
5) The principle of causality (i.e. any event can have a causal impact only on those events that occur later than it, i.e. inside a circle with a radius of no more than $l = cdt$, where dt is the time interval between events);

6) The principle of extremum of action (i.e. the geodesic lines of a curved 4-dimensional space are extremal).
7) The principle of symmetry (i.e., the conditions of non-variability, from which conservation laws follow).
8) The principle of relativity (i.e., the equations include only relative quantities, including time).

Thus, Einstein's vacuum equation (27) turned out to be the quintessence of the entire empirical-epistemological heritage acquired by science by the beginning of the 20th century, i.e. by the time of the creation of GR.

However, these epistemological principles are not enough to use vacuum equations (27), (42) and (46) to construct metric-dynamic models of stable vacuum formations. Therefore, we will formulate three more fundamental ontological principles of the Algebra of Signatures, which are taken from empirically verified philosophical and religious sources.

1] Principle of "Absolute Absence": – "Everything that can appear from emptiness appears in mutually opposite form, so that on average the emptiness remains empty." From the principle of "Absolute absence" follows the condition of "vacuum balance", which was used in all previous articles of the proposed project (Batanov-Gaukhman, 2023a; Batanov-Gaukhman, 2023b; Batanov-Gaukhman, 2023c; Batanov-Gaukhman, 2023d).

2] The principle of "Fair distribution": – "If something can be realized with a certain probability, then it is necessarily realized in a proportion tending to this probability." From the principle of "Fair distribution", in particular, it follows that all possible solutions to the vacuum equation must be taken into account with the appropriate probability.

3] The principle of "Absence of the finite": – "Continuum INFINITY cannot generate the finite, but from the Continuum it is permissible to generate a discrete closed Infinity." From this principle it follows that all metric-dynamic models of stable vacuum formations must be discrete-infinite.

The following question remains open: – "If $T_{ik} = 0$ (i.e. if on the right side of the Einstein-Hilbert equations (14) there are no: density of matter, its motion, pressure and electromagnetic field), then what fills the Universe, and what is source of curvature of the space-time continuum? The answer to this question will be gradually formed below, but now we can answer: "According to the Algebra of Signatures, this world consists of many stable corpuscular vacuum formations of various scales." At the end of this article, Einstein's third vacuum equation is proposed, with the help of which in subsequent articles the corpuscular cosmological model will be presented and answers to many other questions of modern physics will be given.

1.6 *Effect of the principles of "Absolute absence" and "Fair distribution"*

Let us demonstrate the effect of the principles of "Absolute Absence" and "Fair Distribution" using the example of the Friedmann-Lemaître-Robertson-Walker metric (FLRW-metric) (21). There are four main (non-trivial and non-exotic) cases possible at $k = 1$ and $k = -1$ with signatures $(+ - - -)$ and $(- + + +)$

$$ds^{(+)2}_{FLRW1} = c^2dt^2 - e^{-\frac{2ct'}{r}}\left(\frac{dr^2}{1-r^2} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right), \quad (54)$$

$$ds^{(+)2}_{FLRW2} = c^2dt^2 - e^{\frac{2ct'}{r}}\left(\frac{dr^2}{1+r^2} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right), \quad (55)$$

$$ds^{(-)2}_{FLRW1} = -c^2dt^2 + e^{-\frac{2ct'}{r}}\left(\frac{dr^2}{1-r^2} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right), \quad (56)$$

$$ds^{(-)2}_{FLRW2} = -c^2dt^2 + e^{\frac{2ct'}{r}}\left(\frac{dr^2}{1+r^2} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right), \quad (57)$$

where $a(t) = e^{\pm\frac{ct'}{r}}$.

Averaging all these metrics leads to a zero (trivial) metric

$$ds_{12}^{(\pm)2} = \frac{1}{4}\left(ds_{FLRW1}^{(+)2} + ds_{FLRW2}^{(+)2} + ds_{FLRW1}^{(-)2} + ds_{FLRW2}^{(-)2}\right) = 0\cdot c^2dt^2 + 0\cdot dr^2 + 0\cdot d\theta^2 + 0\cdot sin^2\theta d\varphi^2 = 0. \quad (58)$$

This corresponds to the principle of "Absolute absence", from which the condition of vacuum balance follows.

Averaging metrics (54) and (55) over pairs, as well as metrics (56) and (57), we obtain

$$\begin{aligned} ds_{12}^{(+)2} &= \frac{1}{2}\left(ds_{FLRW1}^{(+)2} + ds_{FLRW2}^{(+)2}\right) = c^2dt^2 - \frac{e^{\frac{2ct'}{r}} + e^{-\frac{2ct'}{r}}}{2}\left(\frac{dr^2}{1-r^4} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right) \text{ with signature } (+ - - -) \\ ds_{12}^{(-)2} &= \frac{1}{2}\left(ds_{FLRW1}^{(-)2} + ds_{FLRW2}^{(-)2}\right) = -c^2dt^2 + \frac{e^{\frac{2ct'}{r}} + e^{-\frac{2ct'}{r}}}{2}\left(\frac{dr^2}{1-r^4} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right) \text{ with signature } (- + + +) \end{aligned} \quad (59)$$

where $\frac{e^{\frac{2ct'}{r}} + e^{-\frac{2ct'}{r}}}{2} = \mathrm{ch}\left(\frac{2ct'}{r}\right)$. (60)

In this case, averaging metrics (54) and (57) over pairs, as well as metrics (55) and (56), we obtain

$$\begin{aligned} ds_{12}^{(\pm)2} &= \frac{1}{2}\left(ds_{FLRW1}^{(+)2} + ds_{FLRW2}^{(-)2}\right) = \frac{e^{\frac{2ct'}{r}} - e^{-\frac{2ct'}{r}}}{2}\left(\frac{dr^2}{1-r^4} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right) \text{ with signature } (0 + + +) \\ ds_{21}^{(\pm)2} &= \frac{1}{2}\left(ds_{FLRW2}^{(+)2} + ds_{FLRW1}^{(-)2}\right) = \frac{e^{\frac{2ct'}{r}} - e^{-\frac{2ct'}{r}}}{2}\left(\frac{dr^2}{1-r^4} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right) \text{ with signature } (0 + + +) \end{aligned} \quad (61)$$

where $\frac{e^{\frac{2ct'}{r}} - e^{-\frac{2ct'}{r}}}{2} = s\mathrm{h}\left(\frac{2ct'}{r}\right)$. (62)

Within the Algebra of Signatures, the averaged metrics (59) – (62) (describing the metric-dynamic state of the "external" and "internal" sides of the vacuum, respectively) define a cosmological model, which will be consistently presented in subsequent articles of this project.

2 Solutions of Einstein's first vacuum equation

2.1 Set of metrics-solution of the first vacuum equation

Let's find exact solutions to Einstein's first vacuum equation (42)

$$R_{ik} = 0. \quad (42')$$

This equation is considered in many scientific publications on modern differential geometry and general relativity, for example, in (Landau & Lifshitz, 1971; Vladimirov, 2005; Eddington, 1924; Buchdahl, 1985). However, none of the books and articles known to the author shows the complete set of solutions to this equation or discusses the relationship between these solutions. Therefore, we repeat the solutions to Eq. (42) in sufficient detail.

At the same time, this chapter will demonstrate the general methodology of multilayer geometrized vacuum physics based on the Algebra of Signature.

At this stage of the study, we are interested in stable curvatures and stable vacuum formations, so we will look for stationary (i.e., time-independent) solutions.

Solutions to Eq. (42) for the stationary case are sought in the spherical coordinate system
$(x_0, x_1, x_2, x_3) = (ct, r, \theta, \varphi)$

in the form of metrics:

$ds^{(-)2} = e^{\nu}c^2dt^2 - e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$ with signature (+ − − −), (63)
Or
$ds^{(+)2} = - e^{\nu}c^2dt^2 + e^{\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$ with signature (− + + +), (64)

where ν and λ are the desired functions t and r.

In metric (63), the nonzero components of the metric tensor are equal to

$$g_{00} = e^{\nu}, \quad g_{11} = - e^{\lambda}, \quad g_{22} = - r^2, \quad g_{33} = - r^2 \sin^2\theta, \tag{65}$$

and their contravariant components are equal

$$g^{00} = e^{-\nu}, \quad g^{11} = - e^{-\lambda}, \quad g^{22} = - r^{-2}, \quad g^{33} = - r^{-2}\sin^{-2}\theta. \tag{66}$$

Substituting time-independent components (65) and (66) into Christoffel symbols (6). Next, substituting the obtained Γ^{λ}_{ik} into the Ricci tensor (5), as a result, for the stationary case, three equations are obtained (Landau & Lifshitz, 1971):

$$R_{00} = R_{11} = \nu'' + \nu'^2 + 2\nu'/r = 0, \tag{67}$$

$$R_{22} = e^{-\lambda}(\lambda'/r - 1/r^2) + 1/r^2 = 0, \tag{68}$$

$$R_{33} = e^{-\lambda}(\nu'/r + 1/r^2) - 1/r^2 = 0, \tag{69}$$

$$\nu = - \lambda.$$

Differential equation (67) has three solutions:

$$\nu_1 = \ln(h_1 + h_2/r), \quad \nu_2 = \ln(h_1 - h_2/r), \quad \nu_3 = h_3, \tag{70}$$

where h_1, h_2, h_3 are integration constants. This can be verified by directly substituting each of these solutions into Eq. (67).

Eqs. (68) and (69) also has three solutions:

$$e^{-\lambda} = e^{\nu} = (1 + r_0/r), \quad e^{-\lambda} = e^{\nu} = (1 - r_0/r), \quad e^{-\lambda} = e^{\nu} = 1, \tag{71}$$

where r_0 is the integration constant.

For $h_1 = 1$, $h_2 = r_0$ and $h_3 = 0$, solutions (70) and (71) turn out to be the same for both differential equations (68) and (69).

Substituting three possible solutions (71) into metric (63), we obtain three metrics with the same signature (+ − − −):

$$s_1^{(+)2} = \left(1-\frac{r_o}{r}\right)c^2dt^2 - \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2, \quad (72)$$

$$ds_2^{(+)2} = \left(1+\frac{r_o}{r}\right)c^2dt^2 - \frac{1}{\left(1+\frac{r_o}{r}\right)}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2, \quad (73)$$

$$ds_3^{(+)2} = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2, \quad (74)$$

where r_0 is the radius of the sphere, the meaning of which will be clarified below.

Performing similar operations with the components of the metric tensor from metric (64),

$$g_{00} = -e^{\nu}, \quad g_{11} = e^{\lambda}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\theta,$$

and their contravariant components

$$g_{00} = -e^{\nu}, \quad g_{11} = e^{\lambda}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\theta,$$

we obtain three more metrics that satisfy the first vacuum equation (42), but with the opposite signature (− + + +):

$$ds_1^{(-)2} = -\left(1-\frac{r_o}{r}\right)c^2dt^2 + \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2, \quad (75)$$

$$ds_2^{(-)2} = -\left(1+\frac{r_o}{r}\right)c^2dt^2 + \frac{1}{\left(1+\frac{r_o}{r}\right)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2, \quad (76)$$

$$ds_3^{(-)2} = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2. \quad (77)$$

Note that at $r_0 = 0$, metrics (72) and (73) become metric (74), and metrics (75) and (76) become metric (77).

All metrics (72) – (77) satisfy the first vacuum equation (42), but only the quadratic form (72) is called the Schwarzschild metric, provided $r_0 = r_g = 2GM/c^4$ (where M is the mass of the star or planet).

2.2 Attempts to find other solutions to the first vacuum equation

In article (Batanov-Gaukhman & Cruz, 2024), an attempt was made to find other solutions to Einstein's first vacuum equation (42) in the case of "inverted" components of the metric tensor of the original metrics

$ds_*^{(+)2} = e^{-\nu}c^2dt^2 - e^{-\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$ with signature (+ − − −), (78)

and

$ds_*^{(-)2} = -e^{-\nu}c^2dt^2 + e^{-\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$ with signature (− + + +), (79)

and also, in the case of complex components of the metric tensor of the original metrics

$ds_{**}^{(+)2} = e^{\pm i\nu}c^2dt^2 - e^{\pm i\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$ with signature (+ − − −), (80)

and

$ds_{**}^{(-)2} = -e^{\pm i\nu}c^2dt^2 + e^{\pm i\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$ with signature (− + + +). (81)

In article (Batanov-Gaukhman & Cruz, 2024) it was shown that in the case of the original metrics (78) – (79), as well as (80) – (81), the solutions to Einstein's first vacuum equation (42) remain the same (72) – (77). That is, no other solutions to the first vacuum equation (42) could be found.

2.3 Seventh solution of Einstein's first vacuum equation

In §1, the fundamental principles of the Algebra of Signature were formulated: "Absolute Absence" and "Fair Distribution". We use these principles in relation to 12 solutions (72) – (77) of Eq. (42).

Since there are no initial preferences, each of these metric decisions can be implemented with equal probability P = 1/6. According to the principle of "Fair distribution", it is necessary to assume that all solutions (72) – (77) can be realized simultaneously with the appropriate probability. Therefore, we perform the averaging of these metrics, provided that their centers are combined at $r = 0$. As a result, we obtain a zero (trivial) metric

$$\frac{1}{6}(ds_1^{(+)2}+ds_2^{(+)2}+ds_3^{(+)2}+ds_1^{(-)2}+ds_2^{(-)2}+ds_3^{(-)2}) = 0\cdot c^2dt^2 + 0\cdot dr^2 + 0\cdot d\theta^2 + 0\cdot sin^2\theta d\varphi^2 = 0, \quad (82)$$

with metric tensor components $g_{ik} = 0$.

Metric (82) is the seventh (trivial) solution of the first vacuum equation (42), which can be easily verified by substituting $g_{ik} = 0$ into this equation, resulting in the identity 0 = 0.

From Eq. (82) we can conclude that if metrics (72) – (74) with signature (+ − − −) describe the conditionally "convex" state of vacuum (see Figure 2), and metrics (75) – (77) with signature (− + + +) describe its conditionally "concave" state. Such stable vacuum formations can only appear if their centers are separated in space (Figure 2). Otherwise, they completely compensate for each other's manifestations.

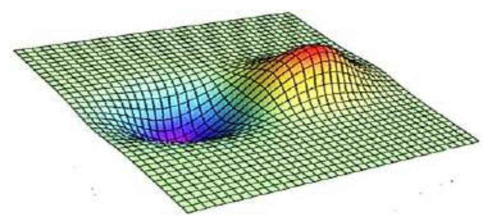


Fig. 2: Two-dimensional illustration of "convexity" and "concavity" separated in space

At the same time, even if the centers of the "convex" and "concave" vacuum formations are in different places, they are completely canceled if averaged over the entire space. This advises the principle of "Absolute absence."

2.4 Coordinate transformation

According to Birkhoff's direct theorem and Israel's inverse theorem, there are no other exact spherically symmetric solutions to the first vacuum equation (42), except for metrics (72) – (78), which at infinity tend to the Minkowski metric (i.e., to the metric of a flat pseudo-Euclidean space).

However, in general relativity, due to the fact that Eq. (42) is generally covariant, there remain many possibilities for choosing other coordinate systems. Of particular interest are coordinate transformations that make it possible to exclude or shift the spatial singularity at $r_0 = r$ in metrics (72) – (73) and (75) – (76).

For example, metric (75)

$$s_1^{(-)2} = -\,(1-\frac{r_0}{r})c^2dt^2 + \frac{1}{(1-r_0/r)}\,dr^2 + r^2(d\theta^2 + sin^2\theta\, d\varphi^2), \quad (75')$$

can be represented in Kruskal-Szekeres coordinates

$$ds_1^{(-)2} = -\frac{4r_0^3}{r(u,v)}e^{-r(u,v)/r_0}\,c^2dudv + r^2(u,v)(d\theta^2 + sin^2\theta d\varphi^2), \quad (83)$$

where $r(u,v)$ is a function that is implicitly defined by the equation

$$\left(1-\frac{r(u,v)}{r_0}\right)e^{-\frac{r(u,v)}{r_0}} = uv. \quad (84)$$

Also, there is no spatial singularity when using Eddington-Finkelstein coordinates. In this case, the Schwarzschild metric (75) takes the form

$$ds_1^{(-)2} = -\left(1-\frac{r_0}{r}\right)c^2dv^2 \pm 2dvdr + r^2(d\theta^2 + sin^2\theta d\varphi^2), \quad (85)$$

where $v = t \pm r^*$, here $(-r^*)$ for a collapsing spherical object (in particular a star); $(+r^*)$ for an expanding (exploding) spherical object;

$$r^* = r + r_0 ln\left|\frac{r}{r_0} - 1\right|. \quad (86)$$

In this case, the time-like singularity has shifted to the center ($r = 0$) of the object under study.

Georges Lemaitre proposed the following transformation of Schwarzschild coordinates $\{t\ ,\ r\}$ into coordinates $\{t\ ,\rho\}$

$$\begin{cases} d\tau = dt + \sqrt{\frac{r_0}{r}}\frac{1}{1-\frac{r_0}{r}}dr, \\ d\rho = dt + \sqrt{\frac{r}{r_0}}\frac{1}{1-\frac{r_0}{r}}d\tau. \end{cases} \quad (87)$$

In these coordinates, for example, metric (72)

$$ds_1^{(+)2} = \left(1-\frac{r_0}{r}\right)c^2dt^2 - \frac{1}{\left(1-\frac{r_0}{r}\right)}\,dr^2 - r^2(d\theta^2 - sin^2\theta d\varphi^2), \quad (88)$$

takes the form

$$ds_1^{(+)2} = c^2d\tau^2 - \frac{r_0}{r}\,d\rho^2 - \left[\frac{3}{2}(\rho - c\tau)\right]^{4/3} r_0^{2/3}(d\theta^2 - sin^2\theta d\varphi^2). \quad (89)$$

In Lemaître coordinates, the singularity also shifted to the middle of the spherically symmetrical object, i.e. to the point $r = 0$. The Lemaître metric (89) is synchronous, i.e. bodies stationary in Lemaître coordinates are in a state of free fall to the central point. Vertically falling bodies reach the gravitational radius $\frac{3}{2}(\rho - c\tau) = r_0$ and the center in a finite proper time.

Allvar Gullstrand in (Gullstrand, 1922) and Paul Painlevé in (Painlev´e, 1921) showed that, for example, the metric (72) can be substituted not in a stationary form, but in a static form with a cross term

$$ds_1^{(+)2} = \left(1-\frac{r_0}{r}\right)c^2dt^2 - 2\sqrt{\frac{r_0}{r}}dtdr - r^2d\theta^2 - r^2\,sin^2\,\theta\,d\phi^2. \quad (90)$$

All solution metrics (72) – (73) and (75) – (76) of the first vacuum equation (42) can be represented in coordinates: Kruskal-Szekeres coordinate; Eddington-Finkelstein coordinates; Lemaître coordinates; Gullstrand-Painlevé coordinates; Isotropic coordinates; Harmonic coordinates.

Behind each of these coordinate systems lies a corresponding process that is subject to separate study, taking into account the methods of the Algebra of Signature, which will be partially outlined in the following paragraphs.

Solutions of the first vacuum equation (42) are sorted into groups that are irreducible to each other. Metrics (72) – (77) belong to different groups, and cannot be converted into each other by any change in the coordinate system.

2.5 Subcont and antisubcont

The main features of two-sided consideration of a 2^3-$\lambda_{m,n}$-vacuum are described in articles (Batanov-Gaukhman, 2023a; Batanov-Gaukhman, 2023b; Batanov-Gaukhman, 2023c; Batanov-Gaukhman, 2023d).

In §7 of article (Batanov-Gaukhman, 2023b) and in §4 and §5 of article (Batanov-Gaukhman, 2023c), the conventional concepts *subcont* (short for "substantial continuum") were introduced to denote the outer 4-dimensional side of the 2^3-$\lambda_{m,n}$-vacuum, and *antisubcont* (short for from "anti-substantial continuum") to refer to the inner 4-dimensional side of the 2^3-$\lambda_{m,n}$-vacuum. These concepts are intended to create the illusion of two continuous environments, *subcont* and *antisubcont* (for example, "white" and "black" colors) for the purpose of convenience of perception of complexly intertwined intra-vacuum processes.

We note once again that the concepts of *subcont* and *antisubcont* are mental (fictional) constructions of two continuous media, which are two 4-dimensional sides of the same extent of 2^3-$\lambda_{m,n}$-vacuum (Batanov-Gaukhman, 2023a; Batanov-Gaukhman, 2023b; Batanov-Gaukhman, 2023c; Batanov-Gaukhman, 2023d). They look like two mutually opposite 4-dimensional ethers (i.e., two elastoplastic media), respectively "white" and "black" in color. However, they should not be perceived as alternatives to two space-time continuums with opposite signatures $(+\,-\,-\,-)$ and $(-\,+\,+\,+)$. It's just that in terms of intertwined continuous elastoplastic media it is much easier to explain the essence of intra-vacuum processes, which will be discussed below.

In accordance with expression (70) in (Batanov-Gaukhman, 2023c), metrics (72) – (74) of the form

$ds^{(+---)2} = g_{ij}^{(+)} dx^i dx^j$ with signature $(+\,-\,-\,-)$

determine the metric-dynamic state of the outer side of the 2^3-$\lambda_{m,n}$-vacuum (i.e. *subcont* is a continuous medium of conventionally "white" color); in this case, metrics (75) – (77) of the form

$ds^{(-+++)2} = g_{ij}^{(-)} dx^i dx^j$ with signature $(-\,+\,+\,+)$

determine the metric-dynamic state of the internal sides of the 2^3-$\lambda_{m,n}$-vacuum (i.e. *antisubcont* is a continuous medium of conventionally "black" color) (see §4 and §5 in (Batanov-Gaukhman, 2023c)).

2.6 Application of the "Absence of the finite" principle

In §9 of article (Batanov-Gaukhman, 2023b) it was shown that any pair of metric 4-spaces with mutually opposite signatures can be represented as a sum (or averaging) of 7 + 7 = 14 metric spaces with other signatures.

For example, a conjugate (i.e., mutually opposite) pair of metrics $ds^{(-++-)2}$ and $ds^{(+--+)2}$ with opposite signatures (− + + −) and (+ − − +) can be expressed by summing (or averaging) 7 + 7 = 14 metric 4-spaces with signatures given in the ranking expression (54) in (Batanov-Gaukhman, 2023b):

(91)

(+ + + +)	+	(− − − −)	= 0
(− − − +)	+	(+ + + −)	= 0
(− − + −)	+	(+ + − +)	= 0
(+ + − −)	+	(− − + +)	= 0
(− + − −)	+	(+ − + +)	= 0
(+ − + −)	+	(− + − +)	= 0
(− + + +)	+	(+ − − −)	= 0
$(- + + -)_+$	+	$(+ - - +)_+$	= 0.

Let's recall that this ranking expression is a consequence of the vacuum balance condition (38) in (Batanov-Gaukhman, 2023b).

Similarly, each mutually opposite pair of metrics with signatures (− + + +) and (+ − − −) from six solutions (72) – (77) can be represented as a summation (or arithmetic averaging) 7 + 7 = 14 metrics with signatures:
Recall that ranking expressions like (99) are a consequence of the vacuum balance condition (38) in (Batanov-Gaukhman, 2023b).

Similarly, each mutually opposite pair of metrics with signatures (− + + +) and (+ − − −) from six solutions (72) – (77) can be represented as a summation (or arithmetic averaging) 7 + 7 = 14 metrics with signatures:

(92)

(+ + + +)	+	(− − − −)	=0
(− − − +)	+	(+ + + −)	=0
(+ − − +)	+	(− + + −)	=0
(− − + −)	+	(+ + − +)	=0
(+ + − −)	+	(− − + +)	=0
(− + − −)	+	(+ − + +)	=0
(+ − + −)	+	(− + − +)	=0
$(+ - - -)_+$	+	$(- + + +)_+$	=0 .

For example, a mutually opposite pair of metrics (72) and (75)

$$ds_1^{(+---)2} = \left(1 - \frac{r_o}{r}\right) c^2dt^2 - \frac{1}{\left(1-\frac{r_o}{r}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 \quad \text{with signature } (+ - - -), \tag{93}$$

$$ds_1^{(-+++)2} = -\left(1 - \frac{r_o}{r}\right) c^2dt^2 + \frac{1}{\left(1-\frac{r_o}{r}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \quad \text{with signature } (- + + +) \tag{94}$$

can be represented as a sum (or averaging) of 7 + 7 = 14 of the same metrics with components

$$g_{00} = (1- r_0/r), \quad g_{11} = (1- r_0/r)^{-1}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\theta, \tag{95}$$

and signatures from rankings (92)

$$\begin{array}{lcll}
ds^{(++++)2} = g_{00}dx_0^2 + g_{11}dx_1^2 + g_{22}dx_2^2 + g_{33}dx_3^2 & + & ds^{(----)2} = -g_{00}dx_0^2 - g_{11}dx_1^2 - g_{22}dx_2^2 - g_{33}dx_3^2 & = 0 \\
ds^{(---+)2} = -g_{00}\,dx_0^2 - g_{11}dx_1^2 - g_{22}dx_2^2 + g_{33}\,dx_3^2 & + & ds^{(+++-)2} = g_{00}dx_0^2 + g_{11}dx_1^2 + g_{22}dx_2^2 - g_{33}dx_3^2 & = 0 \\
ds^{(+--+)2} = g_{00}dx_0^2 - g_{11}dx_1^2 - g_{22}dx_2^2 + g_{33}\,dx_3^2 & + & ds^{(-++-)2} = -g_{00}dx_0^2 + g_{11}dx_1^2 + g_{22}dx_2^2 - g_{33}dx_3^2 & = 0 \\
ds^{(--+-)2} = -g_{00}dx_0^2 - g_{11}dx_1^2 + g_{22}dx_2^2 - g_{33}dx_3^2 & + & ds^{(++-+)2} = g_{00}dx_0^2 + g_{11}dx_1^2 - g_{22}dx_2^2 + g_{33}dx_3^2 & = 0 \\
ds^{(-+--)2} = -g_{00}\,dx_0^2 + g_{11}dx_1^2 - g_{22}dx_2^2 - g_{33}dx_3^2 & + & ds^{(+-++)2} = -g_{00}\,dx_1^2 + g_{11}dx_2^2 + g_{22}dx_2^2 + g_{33}dx_3^2 & = 0 \\
ds^{(+-+-)2} = g_{00}dx_0^2 - g_{11}dx_1^2 + g_{22}dx_2^2 - g_{33}dx_3^2 & + & ds^{(-+-+)2} = -g_{00}dx_0^2 + g_{00}dx_1^2 - g_{22}dx_2^2 + g_{33}dx_3^2 & = 0 \\
ds^{(++--)2} = g_{00}dx_0^2 + g_{11}dx_1^2 - g_{22}dx_2^2 - g_{33}\,dx_3^2 & + & ds^{(--++)2} = -g_{00}dx_0^2 - g_{11}dx_1^2 + g_{22}dx_2^2 + g_{33}dx_3^2 & = 0 \\
\hline
ds^{(+---)2} = g_{00}dx_0^2 - g_{11}dx_1^2 - g_{00}dx_2^2 - g_{00}dx_3^2 & + & ds^{(-+++)2} = -g_{00}\,dx_0^2 + g_{11}\,dx_1^2 + g_{22}dx_2^2 + g_{33}dx_3^2 & = 0
\end{array} \qquad (96)$$

Summation (or averaging) in rankings (92) and (96) is performed by columns (see §9 in (Batanov-Gaukhman, 2023b))

We explain with an example why in the case under consideration addition is equivalent to averaging. Let the denominators of rankings (96) indicate the average of the metrics in the numerator. In this case, the sum of the denominators themselves, according to the vacuum balance condition, is equal to zero

$$\frac{1}{7}\left(g_{00}\ dx_0^2 - g_{11}\ dx_1^2 - g_{00}\ dx_2^2 - g_{00}\ dx_3^2\right) + \frac{1}{7}\left(-g_{00}\ dx_0^2 + g_{11}\ dx_1^2 + g_{00}\ dx_2^2 + g_{00}\ dx_3^2\right) = 0. \qquad (97)$$

Let's multiply both sides of this expression by 7. The result is the denominators in the rankings (96)

$$\left(g_{00}\ dx_0^2 - g_{11}\ dx_1^2 - g_{00}\ dx_2^2 - g_{00}\ dx_3^2\right) + \left(-g_{00}\ dx_0^2 + g_{11}\ dx_1^2 + g_{00}\ dx_2^2 + g_{00}\ dx_3^2\right) = 0. \qquad (98)$$

In turn, conjugate (i.e., mutually opposite) pairs of 4-subspaces from rankings (96) can be decomposed in the same way into sums of 7 + 7 = 14 sub-subspaces, and this can continue indefinitely, if a complete "vacuum balance" is observed (i.e. if the sum of the entire infinite set of mutually exclusive metrics with different signatures is equal to zero).

Thus, when solving the first vacuum equation (42), all three fundamental ontological principles of "Absolute absence", "Fair distribution" and "Absence of the finite" are observed at once.

2.7 Triads of metrics with different signatures

Within the Algebra of Signatures there are additional opportunities to obtain stable vacuum formations.

Let's show this using the example of metric (72)

$$ds_1^{(+---)2} = \left(1 - \frac{r_o}{r}\right)c^2dt^2 - \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2 \text{ with signature } (+---). \qquad (99)$$

This metric can be represented as a sum of three metrics with signatures presented in rankings (see §8 in (Batanov-Gaukhman, 2023b)):

$$\begin{array}{l} (---+) \\ (+-+-) \\ \underline{(++--)} \\ (+---)_+ \end{array} \qquad \begin{array}{l} (--+-) \\ (++--) \\ \underline{(+--+)} \\ (+---)_+ \end{array} \qquad \begin{array}{l} (-+--) \\ (+--+) \\ \underline{(+-+-)} \\ (+---)_+ \end{array} \qquad (100)$$

For example, the first of three rankings (100) is revealed as follows

$$ds_1^{(+)2} = -\left(1-\frac{r_o}{r}\right)c^2dt^2 - \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 - r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2 \quad \text{with signature } (---+) \qquad (101)$$

$$ds_1^{(+)2} = \left(1-\frac{r_o}{r}\right)c^2dt^2 - \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 + r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2 \quad \text{with signature } (+-+-) \qquad (102)$$

$$ds_1^{(+)2} = \left(1-\frac{r_o}{r}\right)c^2dt^2 + \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2 \quad \text{with signature } (++--) \qquad (103)$$

$$ds_1^{(+)2} = \left(1-\frac{r_o}{r}\right)c^2dt^2 - \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2 \quad \text{with signature } (+---) \qquad (104)$$

Similarly, metric (75)

$$ds_1^{(-+++)2} = -\left(1-\frac{r_o}{r}\right)c^2dt^2 + \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2 \text{ with signature } (-+++): \qquad (105)$$

can be presented as a sum of three metrics with signatures presented in rankings:

(106)

(+ + + –)	(+ + – +)	(+ – + +)
(– + – +)	(– – + +)	(– + + –)
(– – + +)	(– + + –)	(– + – +)
(– + + +) +	(– + + +) +	(– + + +) +

For example, the first of three rankings (106) is revealed as follows

$$ds_1^{(-)2} = \left(1-\frac{r_o}{r}\right)c^2dt^2 + \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 + r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2 \quad \text{with signature } (+++-) \qquad (107)$$

$$ds_1^{(-)2} = -\left(1-\frac{r_o}{r}\right)c^2dt^2 + \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 - r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2 \quad \text{with signature } (-+-+)$$

$$ds_1^{(-)2} = -\left(1-\frac{r_o}{r}\right)c^2dt^2 - \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2 \quad \text{with signature } (--++)$$

$$ds_2^{(-)2} = -\left(1-\frac{r_o}{r}\right)c^2dt^2 + \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2 \quad \text{with signature } (-+++)$$

Any of the metrics with signatures (− − − +), (+ − + −), (+ + − −) and (+ + + −), (− + − +), (− − + +), which are in numerators of rankings (105) and (107) is not a solution to the first vacuum equation (42). This can be verified by substituting the components of the metric tensor from these metrics into this equation. However, the sum of triplets of metrics (105) and (107) is equal to either metric (72) with signature (+ − − −), or metric (75) with signature (− + + +), which describe, respectively, more complex stable ones: convexity of *subcont* and the concavity of the *antisubcont*.

There are many combinations of 4-metrics with different signatures from the signature matrix (32) in (Batanov-Gaukhman, 2023b)

$$sign\left(ds^{(a,b)2}\right) = \begin{pmatrix} (++++) & (+++-) & (-++-) & (++-+) \\ (---+) & (-+++) & (--++) & (-+-+) \\ (+--+) & (++--) & (+---) & (+-++) \\ (--+-) & (+-+-) & (-+--) & (----) \end{pmatrix}, \qquad (108)$$

which in sum (or on average) lead to the signature of the Minkowski space (i.e. *subcont*) (+ − − −) and the signature of the anti-Minkowski space (i.e. *antisubcont*) (− + + +). The possibility of application and meaning of these combinations will be revealed in subsequent articles of the proposed project.

2.8 Averaged metric-dynamic state of subcont

2.8.1 Averaging subcont metrics

Let's separately consider three metrics (72) – (74):

$$ds_1^{(+)2} = \left(1-\frac{r_o}{r}\right)c^2dt^2 - \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2, \quad (72')$$

$$ds_2^{(+)2} = \left(1+\frac{r_o}{r}\right)c^2dt^2 - \frac{1}{\left(1+\frac{r_o}{r}\right)}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2, \quad (73')$$

$$ds_2^{(+)2} = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2, \quad (74')$$

which describe the metric-dynamic state of the outer side of the 2^3-$\lambda_{m,n}$-vacuum (i.e. *subcont*).

The third metric (74) is a special case of the first two metrics (72) and (73) for $r_0 = 0$, and describes the state of the original (i.e., uncurved) local section of the *subcont*.

Both metrics (72) and (73) are solutions to the same vacuum equation (42) under the same conditions. There is no reason to prefer either of them, i.e. each of these metrics can be realized with probability ½. Therefore, following the principle of "Fair distribution", we will consider the result of their averaging

$$ds_{12}^{(+)2} = \frac{1}{2}\left(ds_1^{(+)2} + ds_2^{(+)2}\right) = c^2dt^2 - \frac{r^2}{r^2-r_0^2}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2 \quad (109)$$

with averaged components of the metric tensor

$$g_{(12)ik}^{(+)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{r^2}{r^2-r_0^2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The zero component of the metric tensor in the averaged metric (109) is equal to one ($g_{00}^{(+)} = 1$), which means that time t is global.

In a curved 4-dimensional space with a signature (+ − − −), the distance between two events with different r, but with the same other coordinates, is determined by the integral (Landau & Lifshitz, 1971)

$$\xi = \int_{r_1}^{r_2}\sqrt{-g_{11}^{(+)}}\, dr. \quad (110)$$

If $g_{11}^{(+)} = -(1-r_0/r)^{-1}$ from metric (72) or $g_{11}^{(+)} = -(1+r_0/r)^{-1}$ from metric (73) is substituted into integral (110), then such an integral cannot be taken in elementary functions.

By substituting $g_{(12)11}^{(+)} = \frac{r^2}{r^2-r_0^2}$ into the integral (110) from the averaged metric (109), it is possible to find an analytical solution

$$\xi = \int_{r_1}^{r_2} \frac{rdr}{\sqrt{r^2 - r_0^2}} = \sqrt{r^2 - r_0^2}\Big|_{r_1}^{r_2} . \quad (111)$$

Averaging two solutions of the vacuum equation (42) with the same signature (+ − − −) led to a meaningful result.

Let's first find the size of the segment between the points $r_1 = 0$ and $r_2 = r_0$:

$$\sqrt{r^2 - r_0^2}\Big|_0^{r_0} = -\sqrt{-r_0^2} = -\sqrt{-1}r_0 = -ir_0. \quad (112)$$

The length of this segment is equal to the radius of the cavity r_0, and the imaginary nature of this result suggests that the averaged metric (109) does not describe the properties of the *subcont* inside a spherical cavity with radius r_0. In other words, the domain of applicability of metric (109) starts from r_0 and extends to $r_2 = \infty$. In this case we have

$$\sqrt{r^2 - r_0^2}\Big|_{r_0}^{\infty} = \sqrt{\infty^2 - r_0^2} . \quad (113)$$

If the studied *subcont* area were not deformed, then the distance between the points $r_2 = \infty$ and $r_1 = r_0$ would be equal to $r_2 - r_1 = \infty - r_0$, and in our case it is equal to value (113), subtracting one from the other, we find

$$\sqrt{\infty^2 - r_0^2} - (\infty - r_0) = r_0 . \quad (114)$$

since the limit calculation leads to this result

$$\lim_{x\to\infty} \sqrt{x^2 - r_0^2} - (x - r_0) = r_0.$$

The result obtained shows that the *subcont* is compressed by an amount ~ r_0 in all radial directions, and the reason for such compression is due to the fact that it is "displaced" from the cavity with radius r_0. This looks like an air bubble in the liquid (Figure 3).

Fig. 3: Air bubble in liquid

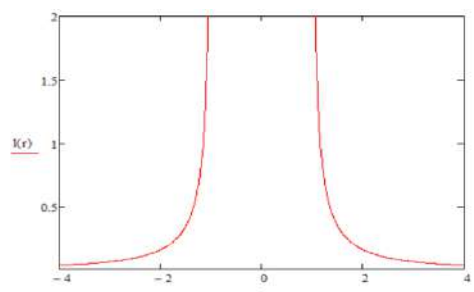


Fig. 4: Graph of a function (117) $l_r^{(+)} = \frac{\Delta r}{r}$

2.8.2 Relative elongation of subcont

We will judge the distortions of the subcont region under study by its relative elongation (see expression (41) in (Batanov-Gaukhman, 2023c))

$$l^{(+)} = \frac{ds^{(+)} - ds_0^{(+)}}{ds_0^{(+)}} = \frac{ds^{(+)}}{ds_0^{(+)}} - 1. \quad (115)$$

In this case, the relative elongation for each coordinate is determined by expressions (47) in (Batanov-Gaukhman, 2023c)

$$l_i^{(+)} = \sqrt{1 + \frac{g_{ii}^{(+)} - g_{ii0}^{(+)}}{g_{ii0}^{(+)}}} - 1, \quad (116)$$

where

$g_{ii}^{(+)}$ are the components of the metric tensor of the curved section of the *subcont*.

$g_{ii0}^{(+)}$ are components of the metric tensor of the same section of the *subcont* before curvature.

Let's substitute into Eq. (116) the components $g_{ii}^{(+)}$ from the averaged metric (109), and the components $g_{ii0}^{(+)}$ from the original metric (74), as a result we obtain

$$l_r^{(+)} = \frac{\Delta r}{r} = \sqrt{\frac{r^2}{r^2-r_0^2}} - 1, \qquad l_\theta^{(+)} = 0, \qquad l_\phi^{(+)} = 0. \tag{117}$$

The graph of the function $l_r^{(+)} = \Delta r/r$, with $r_0 = 1$, is shown in Figure 4. At $r = r_0$, this function tends to infinity $\Delta r/r = \infty$, and at $r < r_0$ it becomes imaginary, which once again confirms the model of an "empty bubble (i.e., a spherical cavity) in a liquid."

Thus, averaging metrics (72) and (73) leads to a metric-dynamic model of a stable (conditionally convex) vacuum formation of the "spherical cavity in a liquid" type, whereas individually these metrics do not lead to such results. This once again confirms that averaging metrics (72) – (73) or (75) – (76)) is not meaningless.

2.8.3 Twisting into subcont k-braids

In §5.2 in (Batanov-Gaukhman, 2023c) it was shown that if two metrics (i.e. quadratic forms) are added (or averaged), in particular

$$ds_{12}^{(+)2} = \frac{1}{2}\left(ds_1^{(+)2} + ds_2^{(+)2}\right), \tag{118}$$

then this corresponds to a segment of a double helix, consisting of two flight lines ("strands") $s_1^{(+)}$ and $s_2^{(+)}$. The segments of these spirals are always mutually perpendicular to each other $ds_1^{(+)} \perp ds_2^{(+)}$ (see Figure 10 in (Batanov-Gaukhman, 2023c)) and can be described by a complex number

$$ds_{12}^{(+)} = \frac{1}{\sqrt{2}}\left(ds_1^{(+)} + i ds_2^{(+)}\right) \tag{119}$$

the squared modulus of which is equal to the averaged metric (118).

Each of these "threads" can consist of two sub-threads $ds_1^{(+)'}$ and $ds_1^{(+)''}$, as well as $s_2^{(+)'}$ and $ds_2^{(+)''}$ (see Figure 10 in (Batanov-Gaukhman, 2023c)). Then the spiral is described by a system of two conjugate complex numbers

$$ds_{12}^{(+)'} = \frac{1}{\sqrt{2}}\left(ds_1^{(+)'} + i ds_2^{(+)'}\right), \tag{120}$$

$$ds_{12}^{(+)''} = \frac{1}{\sqrt{2}}\left(ds_1^{(+)''} - i ds_2^{(+)''}\right),$$

the product of which is also equal to the averaged metric (118).

In accordance with expressions (55) – (59) in (Batanov-Gaukhman, 2023b), the linear elements $ds_1^{(+)}$ и $ds_2^{(+)}$ in metrics (72) and (73) can be represented in the form of spintensors or in the form affine aggregates (i.e. affinors, essentially spirals)

(121)

$$ds_1^{(+)} = \begin{vmatrix} \sqrt{\left(1-\frac{r_o}{r}\right)}cdt + r\sin\theta\, d\phi & \frac{1}{\sqrt{\left(1-\frac{r_o}{r}\right)}}dr + ird\theta \\ \frac{1}{\sqrt{\left(1-\frac{r_o}{r}\right)}}dr - ird\theta & \sqrt{\left(1-\frac{r_o}{r}\right)}cdt - r\sin\theta\, d\phi \end{vmatrix} =$$

$$= \sqrt{\left(1-\frac{r_o}{r}\right)}cdt\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} - \frac{1}{\sqrt{\left(1-\frac{r_o}{r}\right)}}dr\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix} - ird\theta\begin{pmatrix}0 & -i\\i & 0\end{pmatrix} - r\sin\theta\, d\phi\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix};$$

(122)

$$ds_2^{(+)} = \begin{vmatrix} \sqrt{\left(1+\frac{r_o}{r}\right)}cdt + r\sin\theta\, d\phi & \frac{1}{\sqrt{\left(1+\frac{r_o}{r}\right)}}dr + ird\theta \\ \frac{1}{\sqrt{\left(1+\frac{r_o}{r}\right)}}dr - ird\theta & \sqrt{\left(1+\frac{r_o}{r}\right)}cdt - r\sin\theta\, d\phi \end{vmatrix} =$$

$$= \sqrt{\left(1+\frac{r_o}{r}\right)}cdt\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} - \frac{1}{\sqrt{\left(1+\frac{r_o}{r}\right)}}dr\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix} - ird\theta\begin{pmatrix}0 & -i\\i & 0\end{pmatrix} - r\sin\theta\, d\phi\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}.$$

In §5.2 in (Batanov-Gaukhman, 2023c), it was proposed to call the averaged metric of the form (118) a 2-braid.

Thus, according to the classification of the Algebra of Signature, the averaged metric (109) is a 2-braid. in which two lines ("threads") $s_1^{(+)}$ and $ds_2^{(+)}$ are intertwined, defined by affinors (121) and (122), or four twisted sub-lines $ds_1^{(+)'}, ds_2^{(+)'}$, $ds_1^{(+)''}, ds_2^{(+)''}$ (120).

According to §2.6 of this article, each of the metrics (72) and (73) can be represented as a sum of seven sub-metrics with the signatures of the left ranker from the ranking expression (96) with probability 1/7, which, in turn, can be are presented as a sum of sub-sub-metrics with a corresponding probability of 1/49, and such a "deepening" with decreasing probability can continue indefinitely.

Assuming that each sub-metric and sub-sub-metric, etc. defines spiral lines, with a "color" corresponding to their signature (see ranking expression (70) in (Batanov-Gaukhman, 2023c))

(123)

Red	(+ + + +)	+	(– – – –)	Anti-Red
Yellow	(– – – +)	+	(+ + + –)	Anti-Yellow
Orange	(+ – – +)	+	(– + + –)	Anti-Orange
Green	(– – + –)	+	(+ + – +)	Anti-Green
Blue	(+ + – –)	+	(– – + +)	Anti-Blue
Indigo	(– + – –)	+	(+ – + +)	Anti-Indigo
Violet	(+ – + –)	+	(– + – +)	Anti-Violet
White	$(+ - - -)_+$	+	$(- + + +)_+$	Anti-Black

then the results obtained in this paragraph can be illustrated by a two-dimensional "slice" of a 3-dimensional stable vacuum formation of the "spherical cavity in a liquid" type, shown in Figures 4 and 5.

The intertwined fabric of the space-time continuum of the Algebra of Signature is in many ways similar to the spin network of loop quantum gravity.

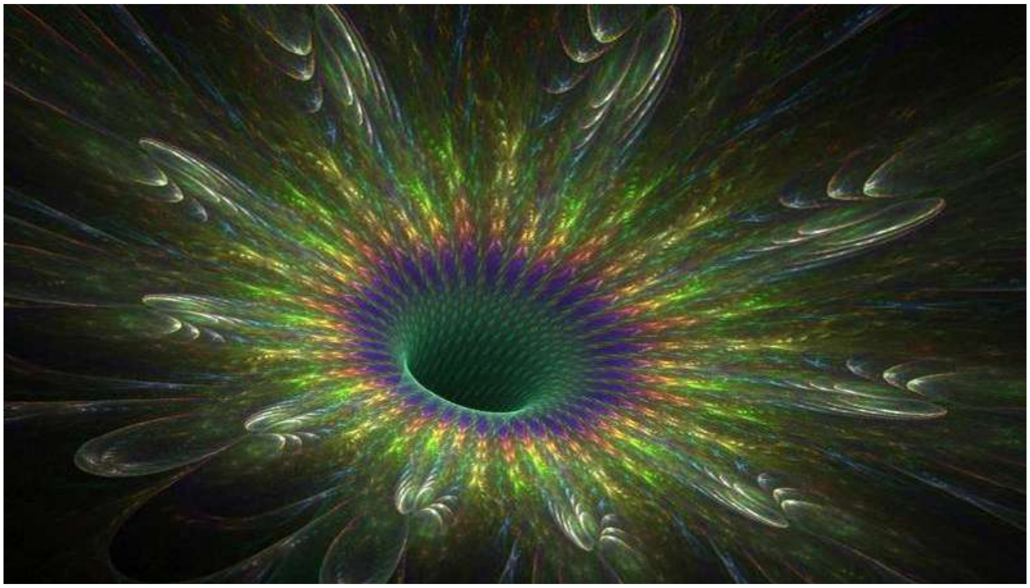


Fig. 5: Fractal illustration of a 2-dimensional slice of a 3-dimensional stable vacuum formation of the "spherical cavity in a liquid" type, consisting of an interweaving of many lines ("threads") of different "colors", which are more and more elongated as they approach a sphere with a radius r_0

2.8.4 Movement of subcont layers (i.e. "white" elastoplastic pseudo-medium)

From the previous paragraph it follows that two metrics (72) and (73) with signature (+ − − −) determine the metric-dynamic state of two 4-dimensional spaces, which are intertwined throughout into a single "fabric" of subcontact.

According to the formal classification of "colors" of the Algebra of Signature (123) (or (70) in (Batanov-Gaukhman, 2023c)), both of these 4-spaces have a white "color", because have a signature (+ − − −). Therefore, for clarity, let us assume that metric (72) describes an elastoplastic pseudo-medium of "a-white" color (or *a-subcont*), and metric (73) describes a pseudo-medium of "*b*-white" color (or *b -subcont*).

Now let's look at how these elastoplastic "pseudo-mediums" move.

In §6.2 in (Batanov-Gaukhman, 2023c), several kinematic cases of motion of layers of double-sided 2^3-$\lambda_{m,n}$-vacuum were considered.

For metrics (72) and (73), the first case is suitable, i.e. metric (91) in (Batanov-Gaukhman, 2023c) with signature (+ − − −)

$$ds^{(+)2} = \left(1 + \frac{v_r^2}{c^2}\right) c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2, \qquad (124)$$

since in this metric, as well as in metrics (72) and (73), the components of the metric tensor $g_{i0}^{(+)} = g_{i0}^{(+)} = 0$.

In turn, metrics (75) and (76) with signature (− + + +) (according to classification (123): elastoplastic pseudo-media of "*a*-black" and "*b*-black" color, i.e. *a-antisubcont* and *b-antisubcont*), corresponds to metric (91) in (Batanov-Gaukhman, 2023c) with a similar signature (− + + +)

$$ds^{(-)2} = -\left(1+\frac{v_r^2}{c^2}\right)c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2. \quad (125)$$

Let's compare $g_{00}^{(+)}$ in metrics (72) and (124), as a result we get

$$1-\frac{r_0}{r} = 1+\frac{v_{r1}^2}{c^2}$$

from where we determine the components of the velocity vector of the "*a*-white" pseudo-medium (i.e. *a-subcont*)

$$-v_{r1}^2 = \frac{c^2r_0}{r} \quad \text{or} \quad v_{r1} = \sqrt{-\frac{c^2r_0}{r}} = i\sqrt{\frac{c^2r_0}{r}} \quad \text{or} \quad -iv_{r1} = \sqrt{\frac{c^2r_0}{r}}, \quad v_{\theta 1} = 0, \quad v_{\phi 1} = 0. \quad (126)$$

Let's compare $g_{00}^{(+)}$ in metrics (73) and (124), as a result we get

$$1+\frac{r_0}{r} = 1+\frac{v_{r2}^2}{c^2},$$

from where we determine the components of the velocity vector of the "*b*-white" pseudo-medium (i.e. *b-subcont*)

$$v_{r2}^2 = \frac{c^2r_0}{r} \quad \text{or} \quad v_{r2} = \sqrt{\frac{c^2r_0}{r}}, \quad v_{\theta 2} = 0, \quad v_{\phi 2} = 0. \quad (127)$$

We also compare $g_{00}^{(+)}$ in the averaged metric (109) and in the metric (124), as a result we obtain for the *subcont* speed on average

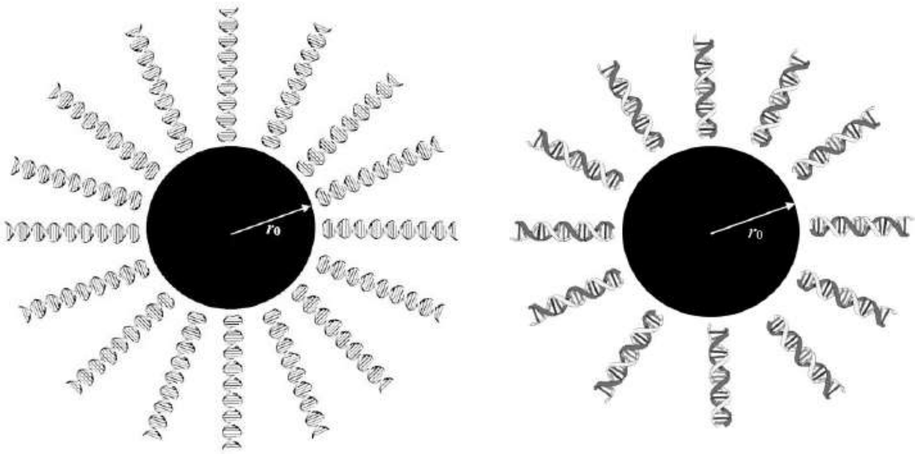
$$1 = 1+\frac{v_r^{(+)2}}{c^2} \quad \text{or} \quad v_r^{(+)2} = 0, \; v_\theta^{(+)} = 0, \; v_\phi^{(+)} = 0. \quad (128)$$

According to Exs. (126), (127) and (128), in all radial directions the average speed of the *ab-subcont* (i.e., the "white" pseudo-medium) is zero

$$v_r^{(+)2} = \frac{1}{2}(v_{r2}^2 - v_{r1}^2) = 0 \quad \text{or} \quad \left|v_r^{(+)}\right| = \frac{1}{2}\left|\sqrt{\frac{c^2r_0}{r}} - i\sqrt{\frac{c^2r_0}{r}}\right| = 0. \quad (129)$$

From Exs. (126), (127) and (129) it is clear that the "*a*-white" pseudo-medium (*a-subcont*) flows in the form of thin streams (currents) from all sides to the edge of the spherical cavity along many spirals, i.e. wrapping around all radial directions (see Figure 6a), and at $r_0 = r$, reaches the speed of light *c*. In this case, the "*b*-white" pseudo-medium (*b-subcont*) flows out in the form of thin streams (currents) from the edge of the spherical cavity in all directions along many spirals (winding around radial directions), starting from the speed of light at $r_0 = r$, and decreasing on the periphery to zero. Taken together, the "*a*-white" and "*b*-white" currents are twisted into opposing double helices (Figure 6a), which, on average, in each local region completely compensate for each other's manifestations. That is, in each local region (outside a spherical cavity with radius r_0) a balance is maintained between inflowing and outflowing currents and countercurrents along "white threads" twisted into double spirals, the relative elongation of which was discussed in §2.8.3.

When extracting information from a set of metrics (72) and (73), we see that the greater the local stretching of the white "threads" of the *subcont*, as we approach the spherical cavity (see Eq. (117) and Figure 4), the greater the speed of the currents flowing along these "threads" (see Exs. (126) and (127) and Figure 6a)).



a)　　　　　*б*)

Fig. 6: a) *a*-white pseudo-medium (*a-subcont*) flows in the form of thin streams (currents) in spirals to the edge of a spherical cavity with radius r_0, gradually increasing the speed from zero to the speed of light c, while *b*-white pseudo-medium (*b-subcont*) flows out in the form of thin streams (currents) in counter-spirals around all radial directions from the edge of a spherical cavity with radius r_0, starting from the speed of light; b) "White" and "black" pseudo-medium flow out and flow in spirals around all radial directions to the edge of a spherical cavity with radius r_0

The speed and acceleration of the a-subcont and b-subcont can be studied not on the basis of the simplified kinematic model (91) in (Batanov-Gaukhman, 2023c), but on a more sophisticated dynamic model presented in §4 in (Batanov-Gaukhman, 2023d). However, it is necessary to devote a separate extensive study to this. Therefore, we will limit ourselves here to only a kinematic consideration.

As was shown in §2.6, each current flowing along the "*a*-white" thread and along the "*b*-white" thread is an interweaving (bundle) of seven sub-currents flowing along sub-threads with "colors" (i.e. with signatures) from the left ranking of expression (123) (or (70) in (Batanov-Gaukhman, 2023c)). In turn, each sub-current is a bundle of 7 sub-sub-currents, and this continues ad infinitum (see §2.6).

The flows of many intertwined "colored" sub-currents along stretched and twisted "threads" are illustrated in Figure 6.

As already noted, "colored" pseudo-environments and "colored" thread-like currents are mental constructions (i.e., a figment of the imagination). Here we have used these concepts only to help thinking understand the essence of mathematical models in terms close to our sensory experience. This is a clear difference in the interpretation of the Algebra of Signatures of zero components of the metric tensor compared to the general relativity of A. Einstein. The zero components of the metric tensor $g_{00}^{(+)}$ *and* $g_{0i}^{(+)} = g_{i0}^{(+)}$ *are here associated not with the change in the flow of time, as in general relativity, but with the movement of pseudo-mediums (see §6 at (Batanov-Gaukhman, 2023c)). The illusion of a moving-deformed continuous medium is more acceptable to our perception than the illusion of a change in the flow of time. The fact is that time is a very complex and multi-valued philosophical category, and we do not know how to measure it. Humanity generally does not have a single instrument capable of measuring time, which is given to us as a sense of duration. Only celestial bodies (planets and stars) allow us to navigate in real time. However, mechanical or electronic watches do not measure time! A watch is a complex technical device that ensures fairly stable rotation of the hands. They allow us to synchronize various processes, but the clocks do not measure practically any time (i.e., the duration we perceive). All such devices are stable synchronizers (i.e., a frequency generator, or a frequency standards) with a certain error. Likewise, we do not measure the real extent of Being with rulers, but only the distance between objects or the sizes of the objects themselves. Ernst Mach loudly declared this in "Mechanics. A historical and critical essay on its development" in*

1883, but not many heard him. Technical synchronizers (which we call clocks or stable frequency standards) can indeed produce clock frequencies differently, depending on whether they are moving relative to the physical vacuum at high speed or not, because these are different versions of their existence, and this may be consistent with the conclusions of relativistic mechanics. However, we note once again that these clocks have practically nothing to do with the complex and unevenly flowing real time (i.e., the duration of Existence). Therefore, it is difficult to perceive the distortion of technical time and the curvature of technical space in the foundations of the fundamental theory. In other words, Minkowski's space-time continuum is a real illusion constructed by the public consciousness. Whereas intuition treats the extension of real Being, as a continuous medium capable of deformation and at the same time moving at an accelerated rate, with great confidence. At the same time, the mathematical model under consideration shows that local deformation of the pseudo-medium is inevitably accompanied by the emergence of a local flow, and vice versa, flows of the pseudo-medium cause its deformations. On the other hand, the interpretation of the results obtained as a 4-curvature of the space-time continuum, or as a 4-distortion of an elastoplastic pseudo-medium, is equivalent in terms of the degree of our confidence in the perception of the surrounding reality. Therefore, the elastoplastic or spatiotemporal interpretation of the calculation results are equivalent and depend on the convenience of their use in solving a particular problem.

At the same time, the elastoplastic interpretation has one undeniable advantage, since in this case, within all areas of metric-dynamic models of vacuum formations and for all vacuum formations included in the general consideration, one global time can be introduced. At the same time, against the background of the global space-time continuum, the parameters of the 4-strain of a continuous elastoplastic pseudo-medium are set everywhere.

2.8.5 Averaged metric-dynamic state of anti-subcount

If with metrics (75) – (77) with signature (– + + +):

$$ds_1^{(-)2} = -\left(1-\frac{r_o}{r}\right)c^2dt^2 + \frac{1}{\left(1-\frac{r_o}{r}\right)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2, \qquad (75')$$

$$ds_2^{(-)2} = -\left(1+\frac{r_o}{r}\right)c^2dt^2 + \frac{1}{\left(1+\frac{r_o}{r}\right)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2, \qquad (76')$$

$$ds_3^{(-)2} = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2 \qquad (77)$$

perform similar actions (109) – (129), then we obtain a metric-dynamic model of exactly the same, but opposite, stable, conditionally "concave" vacuum formation of the "spherical anti-cavity in a liquid" type, with an averaged metric

$$ds_{12}^{(-)2} = \frac{1}{2}\left(ds_1^{(-)2} + ds_2^{(-)2}\right) = -c^2dt^2 + \frac{r^2}{r^2-r_0^2}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2 \qquad (130)$$

and "black" *a-antisubcont* and *b-antisubcont* currents intertwined into bundles.

Above, all known solutions of the Einstein vacuum equation (42) were used and this led to averaged metric-dynamic models of a mutually opposite pair of vacuum formations "spherical cavity" – "spherical anti-cavity". However, the averaged metrics (109) and (130) turned out to be not the Schwarzschild metric, which, according to the entire scientific community, has been reliably tested and experimentally confirmed in the lower orders of approximation of general relativity to Newtonian theory (i.e. for the case of weak gravitational fields). We will show that this problem can be eliminated.

2.8.6 Averaged Schwarzschild metric

Let's return to the consideration of metrics (72) and (73) and assume that in these metrics r_0 differ slightly from each other

$r_{01} \approx r_{02} \approx r_0$ and $r_{01} \geq r_{02}$. (131)

In this case, metrics (72) and (73) take the form

$$ds_1^{(+)2} = \left(1 - \frac{r_{o1}}{r}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{r_{o1}}{r}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2,$$

$$ds_2^{(+)2} = \left(1 + \frac{r_{o2}}{r}\right) c^2 dt^2 - \frac{1}{\left(1 + \frac{r_{o2}}{r}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2.$$

We average these metrics taking into account conditions (131) (i.e., the small difference between r_{01} and r_{02})

$$ds_{12}^{(+)2} = \frac{1}{2}\left(ds_1^{(+)2} + ds_2^{(+)2}\right) \approx \left(1 + \frac{r_{o2} - r_{o1}}{2r}\right) c^2 dt^2 - \frac{r^2}{r^2 - r_0^2} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2. \quad (132)$$

The zero component of the metric tensor in this averaged metric is equal to

$$g_{00} = \left(1 + \frac{r_{o2} - r_{o1}}{2r}\right) = \left(1 - \frac{r_g}{r}\right), \quad (133)$$

where $r_g = \frac{r_{o1} - r_{o2}}{2}$
is a value that can be interpreted as the Schwarzschild radius (or gravitational radius) of a stable corpuscular vacuum formation.

Taking into account expression (133), the averaged metric (132) can be represented in a Schwarzschild-like form

$$ds_{(12)}^{(+)2} \approx \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{1}{1 - \frac{r_0^2}{r^2}} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 \quad (134)$$

For example, it is known that the planet Earth's gravitational radius is approximately equal to $r_{gE} \approx 0.9\ cm$. Then, according to expression (133), if our assumptions are correct, then inside our planet there are two boundary spheres with a difference in radii $r_{o1E} - r_{o2E} \approx 2r_{gE} \approx 1.8$ cm. At the same time, all experimentally confirmed gravitational effects remain in force. It can also be assumed that the average radius of the Earth's solid inner core $r_{0E} \approx 1220$ km.

Thus, within the framework of the considered averaged models of stable vacuum formations, the problem of the Schwarzschild-like gravitational potential is easily solved.

2.8.7 Three possible scenarios for the coexistence of a "spherical cavity" and a "spherical anti-cavity"

The condition for maintaining "vacuum balance" (i.e. the principle of Absolute absence) dictates three main possible scenarios for the coexistence of a "spherical cavity" and a "spherical anti-cavity":

1]. If the cavity (72) – (74) and the anti-cavity (75) – (77) occupy practically the same volume of 2^3-$\lambda_{m,n}$-vacuum (i.e., they are practically combined in coordinates and time) then they completely compensate for each other's manifestations. The most likely scenario for their coexistence is a "dance of death" (see Figures 7 and 8), as a result of which they lose energy in the form of wave disturbances and disappear (annihilate).

2]. If the "cavity" (72) – (74) and the "anti-cavity" (75) – (77) exist simultaneously, but are separated in 3-dimensional space, then, when averaged over the entire space, they also completely compensate for each other's manifestations. Such "convexity" and "concavity" must strive towards each other in order to merge again in the "dance of death".

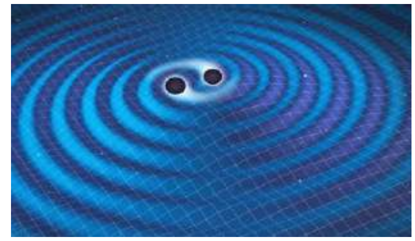


Fig. 7: Illustration of the "cavity" and "anti-cavity" dance of death

3]. "Cavity" (72) – (74) and "anti-cavity" (75) – (77) can be spaced in time, maintaining a "vacuum balance". In this case, they should flow into each other with some periodicity, for example, using the "mechanisms" of explosion and Eddington-Finkelstein collapse (84).

Note that if we strictly adhere to the principle of "Fair distribution", then all three of the above scenarios should be realized with a probability of 1/3.

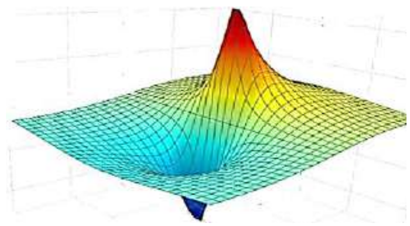


Fig. 8: "Cavity" and "anti-cavity" spaced apart

At this stage of the study, it is not clear what is inside a spherical "cavity" with radius r_0 and in a similar spherical "anti-cavity"? Another problem is this: it turns out that in the world described by Einstein's first vacuum equation (42) there are only two mutually opposite cavities. There is nothing else, but where is the huge variety of entities inhabiting the real world? In addition, the presence of a singularity of the type $l_r^{(+)} = \Delta r/r \to \infty$ raises doubts.

Next, an attempt will be made to answer these questions.

3 Einstein's second vacuum equation

3.1 Averaged second vacuum equation

In (Batanov-Gaukhman & Cruz, 2024), an attempt was made to find additional solutions to Einstein's first vacuum equation (42) in order to solve the problem of filling spherical cavities. However, these studies only strengthened the confidence that such solutions are not contained in Eq. (42). Therefore, we consider solutions to the Einstein vacuum equation with the Λ-term.

Let's consider the system of Einstein vacuum equations (51)

$$\begin{cases} R_{ik} + \Lambda_1 g_{ik} = 0, \\ R_{ik} - \Lambda_2 g_{ik} = 0. \end{cases} \qquad (51')$$

Each of the equations of this system has the right to be applied with a probability of ½; therefore, according to the principle of "Fair distribution", we will look for a solution to the averaged equation

$$R_{ik} + ½\,(\Lambda_1 g_{ik} - \Lambda_2 g_{ik}) = 0,$$

or

$$R_{ik} + ½\, g_{ik}(\Lambda_1 - \Lambda_2) = 0, \qquad (135)$$

where according to Eq. (50)

$$\Lambda_1 = \frac{3}{r_1^2}, \quad \Lambda_2 = \frac{3}{r_2^2}, \quad r_1 \text{ – radius of the first sphere;} \quad r_2 \text{ – radius of the second sphere.} \qquad (136)$$

From the point of view of conservation laws, the averaged vacuum equation (135) has the same properties as any of the equations (51′), because the covariant and ordinary derivatives of the tensor on the left side of this equation are equal to zero.

$$\nabla_j(R_{ik} + ½\,\Lambda_1 g_{ik} - ½\Lambda_2 g_{ik}) = \frac{\partial(R_{ik}+½\,\Lambda_1 g_{ik}-½\Lambda_2 g_{ik})}{\partial x^j} = 0. \quad (137)$$

When considering the vacuum equation (135), three possible cases are identified:

1). If $\Lambda_1 = \Lambda_2$, then Eq. (135) takes the form of the first vacuum equation (42) R_{ik}= 0.

2). If $\Lambda_1 - \Lambda_2 = \pm\Lambda_\Sigma$, then Eq. (135) takes the form of the second vacuum equation

$$R_{ik} \pm ½\,\Lambda_\Sigma\, g_{ik} = 0. \quad (138)$$

3). If $\Lambda_1 - \Lambda_2 = \pm R$, then Eq. (135) takes the form of the Einstein tensor equal to zero

$$R_{ik} \pm ½\, R g_{ik} = 0. \quad (139)$$

This equation, according to expressions (40) – (42) for 4-dimensional space (n = 4), in any case (+) or (–) again takes the form of the first vacuum equation (42).

3.2 Solution of Einstein's second vacuum equation

3.2.1 Metrics-solutions of Kottler - de Sitter - Schwarzschild

The most interesting seems to be the third, self-consistent case, when $\Lambda_1 - \Lambda_2 = \pm R$, but at this stage of the study we only know that Eq. (139) reduces to Einstein's first vacuum equation (42), and its solution have already been discussed in §2 of this article.

Therefore, let us consider solutions to the second vacuum equation (138)

$$R_{ik} \pm \Lambda_a\, g_{ik} = 0, \quad (140)$$

where $\Lambda_a = ½\,\Lambda_\Sigma$, here $\Lambda_a = 3/r_a^2$.

For the stationary, spherically symmetric case, the solutions to Eq. (140) are five metrics with signature (+ – – –)

$$ds_1^{(+)2} = \left(1 - \frac{r_{b1}}{r} + \frac{r^2}{r_{a1}^2}\right) c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_{b1}}{r} + \frac{r^2}{r_{a1}^2}\right)} - r^2(d\theta^2 + sin^2\,\theta\, d\phi^2), \quad (141)$$

$$ds_2^{(+)2} = \left(1 + \frac{r_{b2}}{r} - \frac{r^2}{r_{a2}^2}\right) c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_{b2}}{r} - \frac{r^2}{r_{a2}^2}\right)} - r^2(d\theta^2 + sin^2\,\theta\, d\phi^2), \quad (142)$$

$$ds_3^{(+)2} = \left(1 - \frac{r_{b3}}{r} - \frac{r^2}{r_{a3}^2}\right) c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_{b3}}{r} - \frac{r^2}{r_{a3}^2}\right)} - r^2(d\theta^2 + sin^2\,\theta\, d\phi^2), \quad (143)$$

$$ds_4^{(+)2} = \left(1 + \frac{r_{b4}}{r} + \frac{r^2}{r_{a4}^2}\right) c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_{b4}}{r} + \frac{r^2}{r_{a4}^2}\right)} - r^2(d\theta^2 + sin^2\,\theta\, d\phi^2), \quad (144)$$

$$ds_5^{(+)2} = c^2dt^2 - dr^2 - r^2(d\theta^2 + sin^2\,\theta\, d\phi^2), \quad (145)$$

where $r_{a1}, r_{a2}, r_{a3}, r_{a4}$ and $r_{b1}, r_{b2}, r_{b3}, r_{b4}$ are integration constants (i.e. constant metric parameters) with the dimension of distance.

A system of these metrics determines the stable metric-dynamic state of the *subcont* (i.e., the outer side of the 2^3-$\lambda_{m,n}$-vacuum, see §4 and Figure 7 in (Batanov-Gaukhman, 2023c)).

Also, solutions to Eq. (140) are five metrics with signature (– + + +)

$$ds_1^{(-)2} = -\left(1 - \frac{r_{b1}}{r} + \frac{r^2}{r_{a1}^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_{b1}}{r} + \frac{r^2}{r_{a1}^2}\right)} + r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{146}$$

$$ds_2^{(-)2} = -\left(1 + \frac{r_{b2}}{r} - \frac{r^2}{r_{a2}^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_{b2}}{r} - \frac{r^2}{r_{a2}^2}\right)} + r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{147}$$

$$ds_3^{(-)2} = -\left(1 - \frac{r_{b3}}{r} - \frac{r^2}{r_{a3}^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_{b3}}{r} - \frac{r^2}{r_{a3}^2}\right)} + r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{148}$$

$$ds_4^{(-)2} = -\left(1 + \frac{r_{b4}}{r} + \frac{r^2}{r_{a4}^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_{b4}}{r} + \frac{r^2}{r_{a4}^2}\right)} + r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{149}$$

$$ds_5^{(-)2} = -\,c^2dt^2 + dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2). \tag{150}$$

A system of these metrics determines the stable metric-dynamic state of the *antisubcont* (i.e., the inner side of the 2^3-$\lambda_{m,n}$-vacuum).

Friedrich Kottler first wrote down the Kottler metric of the form (143)

$$ds_{Kottler}^2 = \left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} - r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{143'}$$

in article (Kottler, 1918), which was published in March 1918, almost immediately after the publication of Einstein's general relativity. In the case: $r_a = \infty$ *and* $r_b \neq 0$*, the Kottler metric (143) becomes the Schwarzschild metric*

$$ds_{\text{Schwarzschild}}^2 = \left(1 - \frac{r_b}{r}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_b}{r}\right)} - r^2(d\theta^2 + sin^2\theta\, d\phi^2).$$

In another limiting case: $r_a \neq \infty$ *and* $r_b = 0$*, the Kottler metric (143) becomes the de Sitter metric*

$$ds_{\text{de Sitter}}^2 = \left(1 - \frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r^2}{r_a^2}\right)} - r^2(d\theta^2 + sin^2\theta\, d\phi^2).$$

In the third case: $r_a = \infty$ *and* $r_b = 0$*, the Kottler metric (143) takes the form of the Minkowski metric*

$$ds_{\text{Minkowski}}^2 = c^2dt^2 - dr^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2).$$

Therefore, the metrics-solution (141) – (144) and (146) – (149) of the second Einstein vacuum equation (140) will be called the Kottler - de Sitter- Schwarzschild metrics or, in short, KdSSh-metrics.

3.2.2 The eleventh metric is the solution to the second vacuum equation

Averaging all 10 metrics (141) – (150) with $r_a = r_{a1} = r_{a2} = r_{a3} = r_{a4}$ and $r_b = r_{b1} = r_{b2} = r_{b3} = r_{b4}$ leads to the eleventh zero metric

$$\frac{1}{10}\sum_{i=1}^{10} s_i^2 = 0\cdot c^2dt^2 + 0\cdot dr^2 + 0\cdot d\theta^2 + 0\cdot sin^2\theta\, d\phi^2 = 0, \tag{151}$$

which is also a trivial (zero) solution to the second vacuum equation (140).

3.2.3 Metric-dynamic models of de Sitter space

Let's consider a simplified case when

$$r_{a1} = r_{a2} = r_{a3} = r_{a4} = r_a \quad \text{and} \quad r_b = r_{b1} = r_{b2} = r_{b3} = r_{b4} = 0. \tag{152}$$

Then, when averaging metrics (141) and (144), as well as metrics (142) and (143), only three de Sitter metrics with signature $(+\,-\,-\,-)$ remain:

$$ds_a^{(+)2} = \left(1+\frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1+\frac{r^2}{r_a^2}\right)} - r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{153}$$

$$ds_b^{(+)2} = \left(1-\frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1-\frac{r^2}{r_a^2}\right)} - r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{154}$$

$$ds_c^{(+)2} = c^2dt^2 - dr^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{155}$$

which describe the subcont spherical de Sitter space.

Similarly, when averaging metrics (146) and (149), as well as metrics (147) and (148), only three de Sitter metrics remain with signature $(+\,-\,-\,-)$

$$ds_a^{(-)2} = -\left(1+\frac{r^2}{r_a^2}\right)c^2dt^2 + \frac{dr^2}{\left(1+\frac{r^2}{r_a^2}\right)} + r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{156}$$

$$ds_b^{(-)2} = -\left(1-\frac{r^2}{r_a^2}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{r^2}{r_a^2}\right)} + r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{157}$$

$$ds_c^{(-)2} = -c^2dt^2 + dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2), \tag{158}$$

which describe the antisubkont spherical de Sitter anti-space.

The main capabilities of the Algebra of Signatures (Alsigna) were presented in §2 when constructing a metric-dynamic model of "spherical cavity - anticavity in a liquid" based on a set of metrics (72) – (77).

All these methods are also applicable to the set of metrics (141) – (150). However, we will not repeat here a complete analysis of these metrics, since this can be easily done by analogy with §2, and on the basis of other possibilities of presented in the Algebra of Signatures (Batanov-Gaukhman, 2023a; Batanov-Gaukhman, 2023b; Batanov-Gaukhman, 2023c; Batanov-Gaukhman, 2023d). Let us note only the main features of the solutions to the second vacuum equation.

3.2.4 Spherical de Sitter space

Averaging metrics (153) and (154) leads to the metric (in terms of Alsigna to the *subcontact* 2-braid)

$$ds_{ab}^{(+)2} = c^2dt^2 - \frac{dr^2}{\left(1-\frac{r^4}{r_a^4}\right)} - r^2(d\theta^2 + sin^2\theta\, d\phi^2). \quad (159)$$

The zero component of the metric tensor in the averaged metric (159) is equal to one ($g_{00}^{(+)} = 1$), which means that time t is global.

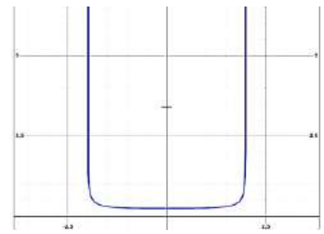


Fig. 9: Graph of the function

Let's substitute the component $g_{11}^{(+)}$ from the metric (159) into the integral (110)

$$\xi = \int_{r_1}^{r_2} \sqrt{-g_{11}^{(+)}}\, dr. \quad (160)$$

As a result, we get

$$\xi = \int_{r_1}^{r_2} \frac{r_a^2}{\sqrt{r_a^4 - r^4}} dr. \quad (161)$$

This integral is not taken in elementary functions, but numerical integration at $r_a = 2$ allows us to obtain the distance function ξ shown in Figure 9.

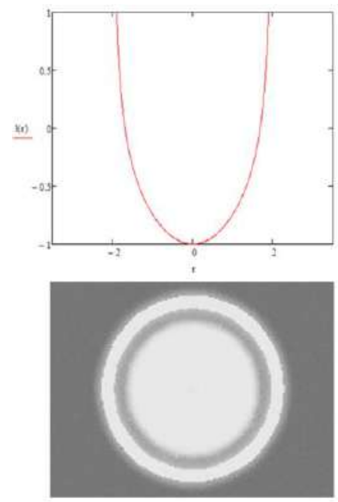


Fig. 10: Graph of the function $l_r^{(+)}$ (120), i.e. relative elongation of the *subcont* in the radial direction.

We substitute the components $g_{ii}^{(+)}$ from the averaged metric (159) and the components $g_{ii0}^{(+)}$ from the metric (155) into the expressions for the relative elongation (116)

$$l_i^{(+)} = \sqrt{1 + \frac{g_{ii}^{(+)} - g_{ii0}^{(+)}}{g_{ii0}^{(+)}}} - 1. \quad (116')$$

As a result, we get

$$l_t^{(+)} = 0, \quad l_r^{(+)} = \frac{\Delta r}{r} = \sqrt{\frac{r_a^4}{r_a^4 - r^4}} - 1, \quad l_\theta^{(+)} = 0, \quad l_\phi^{(+)} = 0. \quad (162)$$

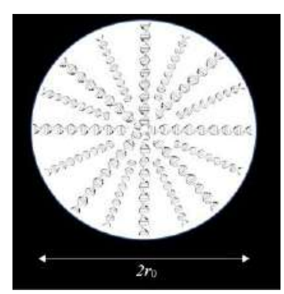


Fig. 11: Radial counter currents of *a-subkont* and *b-subkont*, twisted into double helices

The graph of the function $l_r^{(+)}$, which determines the relative elongation of the *subcont* in the radial direction at $r_a = 2$, is shown in Figure 10. From this graph it is clear that the relative elongation of the *subcont* in the center of such a stable formation is close to (– 1) (i.e., the *subcont* is compressed almost to zero).

Starting from $r = r_a/\sqrt[4]{2}$, as it approaches the periphery of the kernel with radius r_a, the *subcont* is greatly stretched, and at $r = r_a$ its stretch tends to infinity.

Let's compare the zero components $g_{00}^{(+)}$ in metrics (153) and (154) with the zero component in metric (124), as a result we obtain:

- for metric (153)

$$1 + r^2/r_0^2 = 1+ v_{ra}^{(+)2}/c^2 \rightarrow v_{ra}^{(+)2} = c^2r^2/r_0^2 \rightarrow v_{ra}^{(-)} = cr/r_0; \tag{163}$$

- for metric (154)

$$1 - r^2/r_0^2 = 1+ v_{rb}^{(+)2}/c^2 \rightarrow v_{rb}^{(+)2} = -c^2r^2/r_0^2 \rightarrow v_{rb}^{(-)} = - cr/r_0. \tag{164}$$

From Eqs. (163) – (164) it is clear that (by analogy with Exs. (126) – (129) inside the "*subcont* kernel" the *a*-subcont and *b*-subcont currents move towards each other in all radial directions along two threads of the double helices (see Figure 13). Equal in magnitude, but opposite in direction, the radial velocities of the *a-subcont* and *b-subcont* currents $v_{ra}^{(+)} = - v_{rb}^{(+)}$ in the center of the "*subcont* kernel" (i.e. at $r = 0$, see Figure 13) are equal to zero, and at the periphery of this "kernel" with radius r_0 they move at the speed of light.

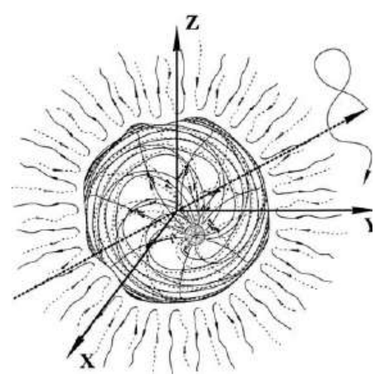
Just as it was shown in §2.8.3 *a-subcont* and *b-subcont* currents consist of sub-currents rolled into bundles, which in turn consist of sub-sub-currents coiled into bundles, and so on until infinity.

The situation seems more physical when, for an external observer, the "*subcont* kernel" rotates. In this case, the *a-subkont* rotates at the periphery of the kernel at the speed of light $v_{ra}^{(+)}(r_0) = c$ (see Figure 12). Then it flows along large spirals with deceleration to the center of the kernel, where $v_{ra}^{(+)}(0) = 0$ practically stops and turns into a *b-subcount*. In turn, the *b-subcont* flows along large spirals from the center of the nucleus with acceleration, starting from the speed $v_{rb}^{(-)}(0) = 0$ and ending with rotation at the periphery of the nucleus at the speed of light $v_{rb}^{(-)}(r_0) = c$ (Figures 11 and 12), where it turns into *a-subcont*. Thus, intrakernel *ab-subcont* "processes" become looped and maintain the highly deformed periphery of the de Sitter's kernel in a stationary state. In this case, the reason for the strong deformation of the *subcont* at the periphery of the core turns out to be associated with centrifugal inertia.

It's like Kurt Gödel's spinning universe, in which centrifugal force balances gravity. Only in the case under consideration, the centrifugal inertia associated with the rotation of the *subcontent* de Sitter kernel opposes the elasticity associated with its deformation. In addition, inside the de Sitter's kernel there is not one general *ab-subcont* rotation around one axis, but simultaneously infinitely many rotations of *ab-subcont* subcurrents (folded into bundles) around many differently directed axes. Therefore, for an external observer, such simultaneous infinite-axial rotation is practically absent.

Thus, the *ab-subcont* currents tied into radial bundles and folded into large spiral arms (see Figure 12) maintain the vacuum balance and stability of the highly deformed interior of de Sitter's kernel.

The subcont de Sitter's kernel, with colossal compression and expansion, turned out to be an extremely difficult place to live. Only in the area of a sphere with radius $r = r_a/\sqrt[4]{2}$ are conditions close to normal and it is possible to survive. Therefore, this kernel is not called the "world".



a)

б)

Fig. 12: a) Model of a rotating "subkont (or anti-subkont) spherical space"; b) Fractal illustration of a rotating "spherical space"

3.2.5 Antisubkont spherical de Sitter space

Averaging metrics (156) and (157) leads to the metric (in terms of Alsigna to the *antisubcont* 2-braid)

$$ds_{ab}^{(-)2} = -c^2dt^2 + \frac{dr^2}{\left(1-\frac{r^4}{r_a^4}\right)} + r^2(d\theta^2 + sin^2\theta\, d\phi^2). \quad (165)$$

Performing actions similar to (159) – (164) with the components of the antisubkont 2-braid (165), we obtain a negative (i.e. completely opposite) stable antisubkont centrally symmetric formation, which we will call the "antisubkont spherical de Sister space".

3.2.6 Annihilation of subcont and antisubcont spherical de Sitter's spaces

Subcont and antisubcont spherical de Sitter spaces completely compensate for each other's manifestations. This is immediately visible, because averaging six metrics (153) – (158) leads to a zero metric of the form (151).

At the same time, the annihilation of *subkont* and *antisubkont* spherical de Sitter spaces can be accompanied by periodic processes.

The coordinate transformation proposed by Lemaitre and Robertson (Vladimirov, 2005)

$$r' = \frac{r}{\sqrt{1+\frac{r^2}{r_a^2}}} r_a e^{-\frac{ct}{r_a}}, \qquad ct' = ct + r_a\, ln\sqrt{1+\frac{r^2}{r_a^2}} \quad (166)$$

leads, for example, a pair of mutually opposite metrics (153) and (156) to the form

$$ds_a^{(-)2} = c^2dt'^2 - e^{-\frac{2ct'}{r_a}}[dr'^2 + r'^2(d\theta^2 + \sin^2\theta\, d\varphi^2)], \quad (167)$$

$$ds_a^{(+)2} = -c^2dt'^2 + e^{\frac{2ct'}{r_a}}[dr'^2 + r'^2(d\theta^2 + \sin^2\theta\, d\varphi^2)]. \quad (168)$$

When averaging these metrics, we obtain a 2-braid

$$ds_{aa}^{(\pm)2} = 0 + \frac{e^{\frac{2ct'}{r_a}} - e^{-\frac{2ct'}{r_a}}}{2}[dr'^2 + r'^2(d\theta^2 + \sin^2\theta\, d\varphi^2)] \text{ with signature } (0+++). \quad (169)$$

This type of averaged metric is associated with the periodic nature of intra-vacuum processes, since the hyperbolic sine

$$\frac{e^{\frac{2ct'}{r_0}} - e^{-\frac{2ct'}{r_0}}}{2} = sh\left(\frac{2ct}{r_0}\right) = -i\, sin\left(i\frac{2ct}{r_0}\right) \quad (170)$$

is a periodic function.

The second pair of mutually opposite metrics (154) and (157), as a result of coordinate transformations

$$r' = \frac{r}{\sqrt{1-\frac{r^2}{r_a^2}}} r_a e^{-\frac{ct}{r_a}}, \quad ct' = ct + r_a \ln \sqrt{1-\frac{r^2}{r_a^2}}, \tag{171}$$

also on average they form a 2-braid

$$ds_{bb}{}^{(\pm)2} = 0 - \frac{e^{\frac{2ct'}{r_a}} - e^{-\frac{2ct'}{r_a}}}{2} [dr'^2 + r'^2(d\theta^2 + \sin^2\theta\, d\varphi^2)] \text{ with signature } (0\ - - -), \tag{172}$$

of a periodic nature.

In this case, we got two exotic metrics (169) and (172), which do not have a time coordinate. This result requires additional understanding.

3.2.7 Compliance with ontological principles

Similar to how it was shown in §2.5, each mutually opposite pair of metrics (153) – (158) can be represented as a sum of 7 + 7 = 14 sub-metrics with the corresponding signatures (as, for example, in rankings (92) или (96). Mutually opposite pairs of sub-metrics, in turn, can be represented as a sum of 7 + 7 = 14 sub-sub-metrics, and so on ad infinitum.

Thus, a set of generalized de Sitter metrics (153) – (158) describe the metric-dynamic state of a stationary spherical space and a stationary spherical anti-space, satisfying all three ontological principles: "Absolute absence", "Fair distribution" and "Absence of finitude."

4 Schwarzschild-de Sitter cell and anti-cell

Let us return to the consideration of metrics (141) – (150) under the condition

$r_{a1} = r_{a2} = r_{a3} = r_{a4} = r_a$ и $r_b = r_{b1} = r_{b2} = r_{b3} = r_{b4} = 0$.

In this case, we have five metric solutions to Eq. (140) with signature (+ − − −)

I $$ds_1^{(+)2} = \left(1 - \frac{r_b}{r} + \frac{r^2}{r_a^2}\right) c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)} - r^2(d\theta^2 + sin^2\,\theta\, d\phi^2), \tag{173}$$

H $$ds_2^{(+)2} = \left(1 + \frac{r_b}{r} - \frac{r^2}{r_a^2}\right) c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} - r^2(d\theta^2 + sin^2\,\theta\, d\phi^2), \tag{174}$$

V $$ds_3^{(+)2} = \left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right) c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} - r^2(d\theta^2 + sin^2\,\theta\, d\phi^2), \tag{176}$$

H′ $$ds_4^{(+)2} = \left(1 + \frac{r_b}{r} + \frac{r^2}{r_a^2}\right) c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_{b4}}{r} + \frac{r^2}{r_a^2}\right)} - r^2(d\theta^2 + sin^2\,\theta\, d\phi^2), \tag{177}$$

i $$ds_5^{(+)2} = c^2dt^2 - dr^2 - r^2(d\theta^2 + sin^2\,\theta\, d\phi^2); \tag{178}$$

and five metric-solutions of the same equation with signature (– + + +)

H′ $$ds_1^{(-)2} = -\left(1-\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)c^2dt^2+\frac{dr^2}{\left(1-\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)}+r^2(d\theta^2+sin^2\theta\, d\phi^2), \quad (179)$$

V $$ds_2^{(-)2} = -\left(1+\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)c^2dt^2+\frac{dr^2}{\left(1+\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)}+r^2(d\theta^2+sin^2\theta\, d\phi^2), \quad (180)$$

H $$ds_3^{(-)2} = -\left(1-\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)c^2dt^2+\frac{dr^2}{\left(1-\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)}+r^2(d\theta^2+sin^2\theta\, d\phi^2), \quad (181)$$

I $$ds_4^{(-)2} = -\left(1+\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)c^2dt^2+\frac{dr^2}{\left(1+\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)}+r^2(d\theta^2+sin^2\theta\, d\phi^2), \quad (182)$$

i $$ds_5^{(-)2} = -\,c^2dt^2+dr^2+r^2(d\theta^2+sin^2\theta\, d\phi^2). \quad (183)$$

Let's average metrics (173) – (177) with signature (+ – – –) and metrics (179) – (182) with signature (– + + +)

$$ds_{1\text{-}4}{}^{(+)2} = \frac{1}{4}\left(ds_1{}^{(+)2}+ds_2{}^{(+)2}+ds_3{}^{(+)2}+ds_4{}^{(+)2}\right). \quad (184)$$

$$ds_{1\text{-}4}{}^{(-)2} = \frac{1}{4}\left(ds_1{}^{(-)2}+ds_2{}^{(-)2}+ds_3{}^{(-)2}+ds_4{}^{(-)2}\right). \quad (185)$$

As a result, we obtain averaged metrics

$$ds_{1-4}^{(+)2} = c^2dt^2-g_{11}^{(+)}(r)dr^2-r^2(d\theta^2+sin^2\theta\, d\phi^2), \quad (186)$$

$$ds_{1-4}^{(-)2} = -\,c^2dt^2+g_{11}^{(-)}(r)dr^2+r^2(d\theta^2+sin^2\theta\, d\phi^2), \quad (187)$$

where

$$g_{11}^{(+)}(r) = g_{11}^{(-)}(r) = \frac{1}{4}\left[\frac{1}{\left(1-\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)}+\frac{1}{\left(1+\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)}+\frac{1}{\left(1-\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)}+\frac{1}{\left(1+\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)}\right]. \quad (188)$$

Let us substitute the components $g_{ii}^{(+)}$ of the averaged metric (186) or the components $g_{ii}^{(-)}$ of the averaged metric (187) into the expressions for the relative elongation (116)

$$l_i^{(+)} = \sqrt{1+\frac{g_{ii}^{(+)}-g_{ii0}^{(+)}}{g_{ii0}^{(+)}}}-1, \qquad l_i^{(-)} = \sqrt{1+\frac{g_{ii}^{(-)}-g_{ii0}^{(-)}}{g_{ii0}^{(-)}}}-1,$$

where the components $g_{ii0}^{(+)}$ are taken from the undistorted metric (178), and the components $g_{ii}^{(-)}$ are taken from the undistorted metric (183).

As a result, we get

$$l_r^{(\pm)} = \frac{\Delta r}{r} = \sqrt{g_{11}^{(\pm)}(r)} - 1 = \sqrt{\frac{1}{4}\left[\frac{1}{\left(1-\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)} + \frac{1}{\left(1+\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)} + \frac{1}{\left(1-\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)} + \frac{1}{\left(1+\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)}\right]} - 1, \qquad (189)$$

$$l_t^{(\pm)} = 0, \qquad l_\theta^{(\pm)} = 0, \qquad l_\phi^{(\pm)} = 0.$$

The graph, for example, of the function $l_r^{(+)}$ (189) with $r_a = 60$ and $r_b = 1{,}5$, which determines the relative elongation of vacuum in the radial direction, is shown in Figure 13. From this graph it is clear that the result is an almost hollow ball (i.e., a spherical de Sitter space) with compacted edges, inside of which there is a spherical Schwarzschild cavity, which is described by metrics (16) – (18), more precisely, by the averaged metric (25).

Indeed, if in metrics (173) – (177) we direct r_a to infinity ($r_a \to \infty$), i.e., for example, assume that r_a is the radius of the Universe, then in the vicinity of a small cavity with radius $r_{b,} = r_0$, which is commensurate, for example, with the gravitational radius of the "black hole", the deformed state of the vacuum will be described by the averaged metric (25).

Shown in Figure 13, the vacuum formation resembles a biological cell with an outer shell and an internal nucleolus, so we will call this formation a Schwarzschild-de Sitter cell.

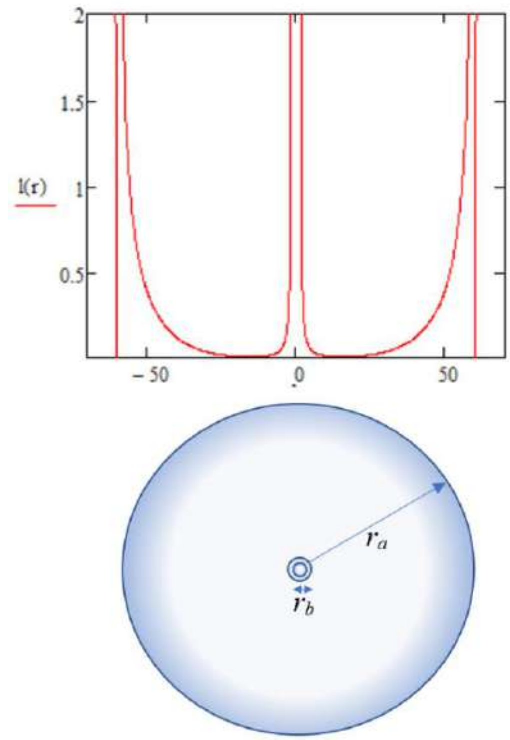


Fig. 13: Graph of the relative elongation function $l_r^{(+)}$ (189), which determines the relative elongation of vacuum in the radial direction

Performing similar operations with decision metrics (179) – (183) with the opposite signature (− + + +), we obtain exactly the same, but opposite Schwarzschild - de Sitter anti-cell.

If we conventionally accept that the Schwarzschild - de Sitter cell is a "convexity" of the vacuum, then the Schwarzschild - de Sitter anti-cell is its "concavity".

Averaging all ten metrics-solutions (173) – (183) of the second vacuum equation (140) leads to two trivial (i.e. zero) pseudo-metric-solutions of this equation

$$\frac{1}{10}\left(\sum_{k=1}^{5} ds_k^{(+)2} + \sum_{k=1}^{5} ds_k^{(-)2}\right) = \pm 0 \cdot c^2 dt^2 \mp 0 \cdot dr^2 \mp 0 \cdot r^2 d\theta^2 \mp 0 \cdot r^2 \sin^2\theta\, d\phi^2. \qquad (190)$$

It is obvious that Einstein's vacuum equation with the Λ-term (140) also does not allow us to solve the problem of filling the "spherical Schwarzschild cavity" and the "spherical anti-Schwarzschild cavity", which in this case find themselves inside the de Sitter space or the anti-de Sitter space, respectively.

We compare the zero components $g_{00}^{(+)}$ in metrics (173) – (178) with the zero component in metric (124)

$$ds^{(+)2} = \left(1 + \frac{v_r^2}{c^2}\right) c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2. \qquad (124')$$

As a result, we obtain the velocities of four *subcont's* currents intertwined into bundles

$$\left(1+\frac{v_{r1}^2}{c^2}\right)=\left(1-\frac{r_b}{r}+\frac{r^2}{r_a^2}\right) \rightarrow v_{r1}^2=\left(\frac{r^2}{r_a^2}-\frac{r_b}{r}\right)c^2 \rightarrow v_{a1}=\sqrt{\frac{r^2c^2}{r_a^2}-\frac{r_bc^2}{r}}=c\sqrt{\frac{r^2}{r_a^2}-\frac{r_b}{r}}, \quad (191)$$

$$\left(1+\frac{v_{r2}^2}{c^2}\right)=\left(1+\frac{r_b}{r}-\frac{r^2}{r_a^2}\right) \rightarrow v_{r2}^2=\left(-\frac{r^2}{r_a^2}+\frac{r_b}{r}\right)c^2 \rightarrow v_{r2}=\sqrt{-\frac{r^2c^2}{r_a^2}+\frac{r_bc^2}{r}}=c\sqrt{\frac{r_b}{r}-\frac{r^2}{r_a^2}},$$

$$\left(1+\frac{v_{r3}^2}{c^2}\right)=\left(1-\frac{r_b}{r}-\frac{r^2}{r_a^2}\right) \rightarrow v_{r3}^2=\left(-\frac{r^2}{r_a^2}-\frac{r_b}{r}\right)c^2 \rightarrow v_{r3}=\sqrt{-\frac{r^2c^2}{r_a^2}-\frac{r_bc^2}{r}}=ic\sqrt{\frac{r^2}{r_a^2}+\frac{r_b}{r}},$$

$$\left(1+\frac{v_{r4}^2}{c^2}\right)=\left(1+\frac{r_b}{r}+\frac{r^2}{r_a^2}\right) \rightarrow v_{r4}^2=\left(\frac{r^2}{r_a^2}+\frac{r_b}{r}\right)c^2 \rightarrow v_{r4}=\sqrt{\frac{r^2c^2}{r_a^2}+\frac{r_bc^2}{r}}=c\sqrt{\frac{r^2}{r_a^2}+\frac{r_b}{r}}.$$

Since v_{ri} cannot exceed the speed of light, the conditions must be met

$$0\le\frac{r^2}{r_a^2}+\frac{r_b}{r}\le 1,\quad 0\le\frac{r^2}{r_a^2}-\frac{r_b}{r}\le 1,\quad 0\le\frac{r_b}{r}-\frac{r^2}{r_a^2}\le 1.$$

From these expressions it is clear that in this case, two *subcont's* currents flow out in spirals from the periphery of the *world* at the speed of light, as described in §3.2.4. Then they slow down. However, near the inner nucleolus they again accelerate to the speed of light and turn into two opposite *subcont's* currents, which, along the same "threads" twisted in a spiral, return to the periphery of the Schwarzschild-de Sitter cell, first slowing down and then accelerating to the speed of light.

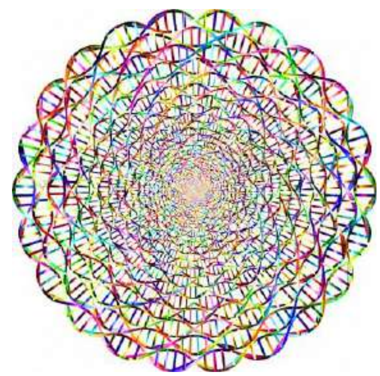


Fig. 14: Illustration of the interweaving of "threads" (i.e. lines) of several affine subspaces forming the "fabric" of a two-sided 2^3-$\lambda_{m,n}$-vacuum

Average *subcont's* speed in each local region of the Schwarzschild-de Sitter cell

$$v_r^{(+)2}=\frac{1}{4}\left[\left(1-\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)+\left(1+\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)+\left(1-\frac{r_b}{r}-\frac{r^2}{r_a^2}\right)+\left(1+\frac{r_b}{r}+\frac{r^2}{r_a^2}\right)\right]=0. \quad (192)$$

This means that the inflowing and outflowing currents, twisted in a 4-helix, completely compensate for each other's manifestations, ensuring *subcont's* balance and stability of the *subcont's* deformations shown in Figure 13.

5 Deepening model concepts to infinity

5.1 Infinitely intertwined fabric of $\lambda_{m,n}$-vacuum

Everything that was said in §2 and §3 in relation to infinite metric-dynamic models of "spherical cavities" also concerns the Schwarzschild-de Sitter cell and anti-cell.

In the case under consideration, the two-sided metric space (i.e., 2^3-$\lambda_{m,n}$-vacuum) is the result of the superposition of eight metric spaces (173) – (177) and (179) – (190) or interweaving of 16 affine extensions, which belong to 8 + 8 = 16 linear forms twisted into 16-braid.

In Figure 14 and 15 shows an illustration of the interweaving of several affine subspaces forming a two-sided metric space.

The properties of intertwined affine sub-spaces and multilayer metric spaces with signatures (+ − − −) and (− + + +), corresponding to the "vacuum balance" condition (+ − − −) + (− + + +) = 0, are described in detail in the "Algebra of Signatures" (Batanov-Gaukhman, 2023a; Batanov-Gaukhman, 2023b; Batanov-Gaukhman, 2023c; Batanov-Gaukhman, 2023d).

Fig. 15: Fractal illustration of the intertwined "fabric" of a double-sided 2^3-$\lambda_{m,n}$-vacuum

Depth-infinite metric-dynamic models of stable $\lambda_{m,n}$-vacuum formations of the type "spherical cavity" (§2.6), de Sitter spherical space and anti-space (§3.2.4), and Schwarzschild - de Sitter cells and anti-cells (§4), separate extensive studies can be devoted, taking into account various coordinate transformations, for example (83), (87), (90), etc. But all these models, infinite in depth, based on solutions of Einstein's first and second vacuum equations, describe single stable vacuum objects.

Therefore, the general question remains open: "How to introduce a model idea of the huge variety of spherical formations that fill the reality around us?

5.2 Qualitative discussion of the singularity problem

At $r = r_0$, the relative vacuum elongation functions (117), (162) and (189) tend to infinity ($\Delta r/r \to \infty$, see Figures 4, 10, 13) - this is a clear indicator of the incompleteness of the mathematical model under consideration.

It is obvious that within the framework of Riemann differential geometry, the problem of the presence of singularities in the metrics-solutions of the vacuum equations (42) and (140) is in principle unsolvable. Perhaps this problem will be solved as a result of increasing the capabilities of differential geometry, for example, by taking into account not only curvature, but also torsions, displacements and other distortions of space.

In other words, solving the problem of singularities requires a radical increase in the capabilities of the mathematical apparatus of differential geometry.

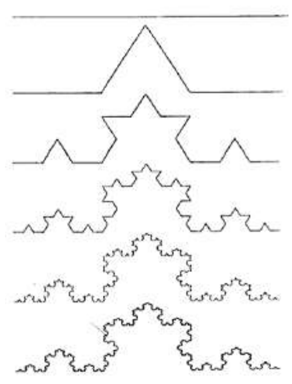


Fig. 16: The first six iterations of the Koch curve fractal

In this article we note only one circumstance that can help solve this problem.

Let us recall the property of the "Koch curve" fractal (Figure 16).

This fractal has two extraordinary properties: 1) any iteration of the Koch curve is an example of a continuous line to which it is impossible to draw a tangent at any point (i.e., these lines are not differentiable); 2) if the length of the initial Koch segment is 1, then the length of the n-th iteration of this fractal is equal to $(4/3)^{n-1}$, therefore the length of the Koch curve at $n = \infty$ tends to infinity.

Let's return to the problem of singularities in averaged metrics (109), (130), (134), (186) and (187). It should be expected that in the region of a sphere with radius r_0 (this region is called "*raqiya*"), an increase in the length of radial segments along the length of the vacuum occurs with a decrease in the scale of their kinks (see Figure 17), similar to a decrease in the scale of kinks in the "Koch curve" as the number of iterations (see Figure 16). Moreover, as we approach r_0, the elongation of such broken, or bent, or wound, etc., segments can tend to infinity.

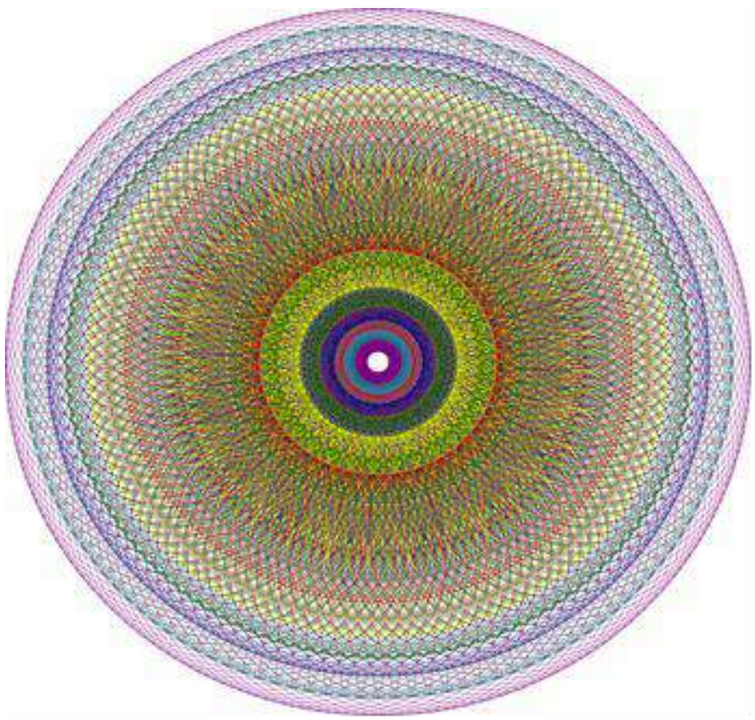
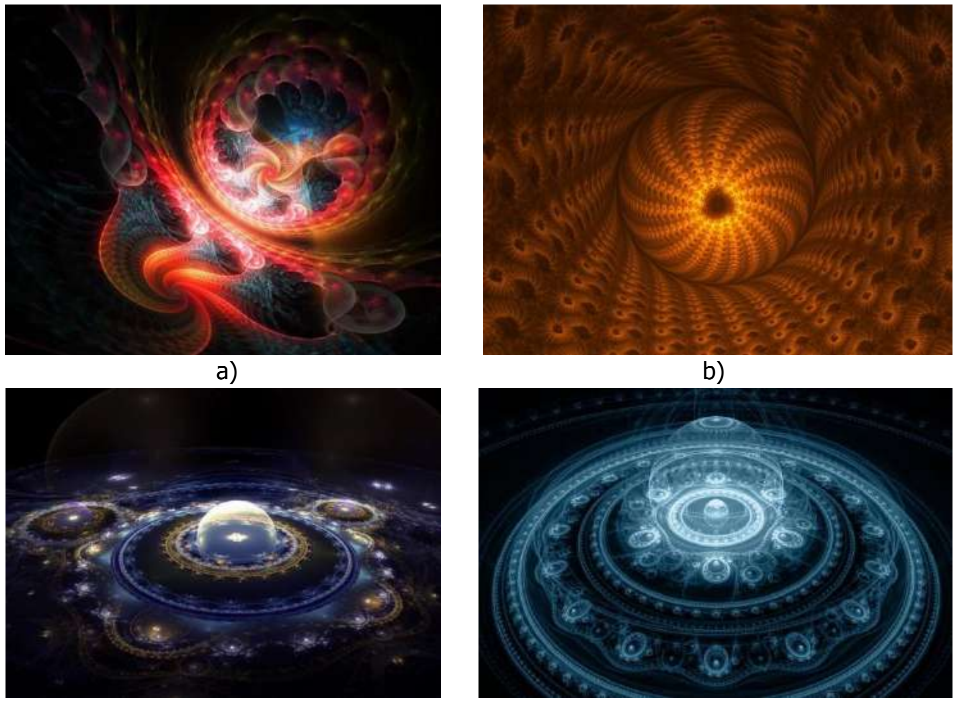


Fig. 17: Increase in brokenness of lines as they approach the central cavity (Prokhorov-Lebedev drawing)

In Figure 18 shows fractal illustrations of complexly curved "*raqiya*" (i.e., shell) around the core of the vacuum formation.



a) b) c) d)

Fig. 18: Fractal illustrations of a complexly curved region of vacuum (i.e., "*raqiya*") surrounding the core of a corpuscular formation with radius r_0. Fractals c and d show two or more ring-shaped boundaries surrounding a spherical core. Benoit Mandelbrot in the book "Fractal Geometry of Nature" noted that recursive algorithms of fractal geometry have the phenomenon of visualizing the meaning and properties of natural objects

6 Einstein's third vacuum equation

As shown above, solutions to the first and second Einstein vacuum equations (42) and (140) make it possible to construct metric-dynamic models of a mutually opposite pair of single stable vacuum formations, but do not allow solving the problem of filling spherical cavities and anti-cavities inside these formations. In addition, these equations lack the potential to describe many stable spherical objects. In this regard, it is proposed to consider the possibility of expanding the vacuum equation (140).

Let's recall that in order to write down Eq. (25), Einstein used the following property of the metric tensor

$$\Lambda\nabla_j g_{ik} = \nabla_j \Lambda g_{ik} = 0.$$

However, it is obvious that the covariant derivative of the infinite series of Λ_i-terms are also equal to zero

$$\nabla_j(\Lambda_1 g_{ik} + \Lambda_2 g_{ik} + \Lambda_3 g_{ik} + \ldots + \Lambda_\infty g_{ik}) = \Lambda_1 \nabla_j g_{ik} + \Lambda_2 \nabla_j g_{ik} + \ldots + \Lambda_\infty \nabla_j g_{ik} = 0, \quad (193)$$

where Λ_1, Λ_2, … , Λ_∞ are constants that can take both positive ($\Lambda_i > 0$) and negative ($\Lambda_j < 0$) values.

We use the same method that Einstein used to introduce the Λ-term into Eq. (25), and write the vacuum equation

$$R_{ik} - \frac{1}{2} R g_{ik} + \Lambda_1 g_{ik} + \Lambda_2 g_{ik} + \Lambda_3 g_{ik} + \ldots + \Lambda_\infty g_{ik} = 0, \quad (194)$$

where according to Eq. (50) $\Lambda_k = 3/r_{aj}^2$ or $-3/r_{aj}^2$, here r_{aj} is the radius of the j-th spherical formation.

The covariant and ordinary partial derivatives of the tensor on the left side of Eq. (194) are equal to zero:

$$\nabla_j(R_{ik} - \frac{1}{2}Rg_{ik} + g_{ik}\sum_{k=1}^{\infty}\Lambda_k) = \frac{\partial(R_{ik} - \frac{1}{2}Rg_{ik} + g_{ik}\sum_{k=1}^{\infty}\Lambda_k)}{\partial x^j} = 0, \qquad (195)$$

therefore, this equation is an expression of conservation laws, just like the first and second vacuum equations (42) and (140).

Eq. (194) will be called Einstein's third vacuum equation.

In the following articles of the proposed project "Geometrized vacuum physics based on the Algebra of Signature" it will be shown that solutions to Eq. (194) allow the description of a multitude of interacting stable vacuum formations of different sizes. This will allow us to develop the vacuum theory of elementary particles and propose a corpuscular cosmological model.

CONCLUSION

In this fifth part of the scientific project under the general title "Geometrized physics of vacuum based on the Algebra of Signature", metric-dynamic models of stable $\lambda_{m,n}$-vacuum formations are considered, based on solutions of Einstein's first vacuum equation (42) and the second vacuum equation (140).

As a result, with simplifications related to Riemann geometry, from the solutions of the first and second Einstein vacuum equations, it was possible to construct three types of metric-dynamic models of stable $\lambda_{m,n}$-vacuum formations: "spherical cavity and anti-cavity" (72) – (77), "de Sitter spherical space and anti-space" (153) – (158) and "Schwarzschild-de Sitter cell and anti-cell" (173) – (183).

The construction of these models was carried out on the basis of three newly introduced ontological principles of "Absolute Absence", "Fair Distribution" and "Finite Absence" (see §1.5), as well as using the Signature Algebra developed in the first four articles of the proposed project (Batanov-Gaukhman, 2023a; Batanov-Gaukhman, 2023b; Batanov-Gaukhman, 2023c; Batanov-Gaukhman, 2023d).

As a result, it is shown that averaging a complete or partial set of metrics-solutions to Einstein's vacuum equations leads to other solutions to these equations and/or to reasonable results that are not achievable when considering each metric-solution separately.

However, as noted by mathematician David Reid, it is possible that useful information may be contained not only in the arithmetic mean of metrics-solutions of vacuum equations, but also in other types of averaging, for example in their: geometric mean, or harmonic mean, or square mean, or cubic mean.

The zero component of the metric tensor of all averaged metrics (109), (130), (134) и (189) obtained in this article is equal to one ($g_{00}^{(+)} = 1$), i.e. time t in these metrics is global. This means that the stable vacuum formations described by these metrics can coexist in the same global space with the same time. Global time t in these metrics can be synchronized with the universal time of averaged cosmological metrics with zero components $g_{00}^{(+)} = 1$ and $g_{00}^{(-)} = -1$ (59) and (60). Therefore, such stable vacuum formations can indeed be synchronized (i.e., "frozen") into the space of the cosmological model that changes with time t.

For convenience of perception of multilayer intra-vacuum processes, this article has changed the interpretation of the components of the metric tensor and metrics-solutions of vacuum equations in general. If in the general relativity metrics (i.e. quadratic forms) characterize the curvature of the space-time continuum, and zero components are associated with changes in the flow of time, then in the proposed theory the metric characterizes the deformations of a continuous elastic-plastic pseudo-substantial medium, and the zero components are related to the speed of its movement.

Another serious difference between the theory developed here and general relativity is due to the fact that in Geometrized Vacuum Physics at least two sets of metrics with mutually opposite signatures (+ − − −) and (− + + +) are used, and upon a more detailed consideration all 16 signatures (108). This makes it possible to constantly maintain a "vacuum balance", and, ultimately, leads to the solution of many problems, including the problem of "baryonic asymmetry of the Universe".

The infinitely deepening intertwined fabric of the space-time continuum of Alsigna taking into account all 16 signatures (108) (i.e., 16 types of topologies, see §4 in (Batanov-Gaukhman, 2023b)) is in many ways similar to the spin network of loop quantum gravity and 6-dimensional Callabi-Yau manifolds (see §9 in (Batanov-Gaukhman, 2023b)). In this sense, the Algebra of Signatures can serve as a link that unites different directions of development of quantum gravity.

All metric-dynamic models discussed in this article describe only one pair of mutually opposite stable objects of the corpuscular type. That is, it turned out that for stationary cases, the first and second Einstein vacuum equations in their potency contain the possibility of describing only one mutually opposite pair, capable of coexisting only in a state of "dance of death". However, within the framework of the proposed theory, the task is to develop a model representation of the surrounding reality, filled with an infinite number of corpuscles of various sizes, therefore the extended Einstein's third vacuum equation (194) is proposed. The solutions to this equation and the corpuscular cosmological model based on these solutions will be presented in subsequent articles of the proposed project under the general title "Geometrized vacuum physics based on the Algebra of Signatures."

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